Theory and Practice of Geographic Routing

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Abstract Geographic routing algorithms use position information for making packet forwarding decisions. Unlike topological routing algorithms, they do not need to exchange and maintain routing information and work nearly stateless. This makes geographic routing attractive for wireless ad hoc and sensor networks. Most geographic routing algorithms use a greedy strategy that tries to approach the destination in each step, e.g. by selecting the neighbor closest to the destination as a next hop. However, greedy forwarding fails in local minimum situations, i.e. when reaching a node that is closer to the destination than all its neighbors. A widely adopted approach to solve this situations is planar graph routing, which guides the packet around the local minimum and guarantees delivery, required that a planar subgraph of the network graph can be constructed. This chapter gives an overview of the fundamentals of geographic routing, reviews theoretical results and presents new developments towards practical applicability.

1 Introduction

Geographic routing (also known as position-based routing or geometric routing) is a technique to deliver a message to a node in a network over multiple hops by means of position information. Routing decisions are not based on network addresses and routing tables; instead, messages are routed towards a destination location. With knowledge of the neighbors' location, each node can select the next hop neighbor that is closer to the destination, and thus advance towards the destination in each step. The fact that neither routing tables nor route discovery activities are necessary makes geographic routing attractive for dynamic networks such as wireless ad hoc and sensor networks. In such networks, acquiring and maintaining routing information is costly as it involves additional message transmissions that require energy and bandwidth and frequent updates in mobile and dynamic scenarios. In contrast, there are geographic routing algorithms that work nearly stateless and can provide high message delivery rates under mobility. All this applies under the following assumptions:

1. A node can determine its own position.
2. A node is aware of its neighbors' positions.
3. The position of the destination is known.

With GPS or other satellite based navigation systems, position information can be made available to even small mobile devices. Further location systems for indoor applications are described in [37]. The second assumption requires broadcasting the position information locally to other participants in the network. With this information a node is able to determine the next hop that is closer to the destination. In Section 4 we will see that there are even algorithms that do not need neighborhood information in advance. The third assumption can be met by means of a location service that maps network addresses to geographic locations. Section 7 reviews some of these approaches. In some cases, the destination is inherently known to the nodes, e.g. in some sensor network applications where a single sink node collects all the data measurement information.

The main prerequisite to meet the three assumptions is a positioning system. If this is available, geographic routing provides an efficient and scalable solution for routing in wireless and mobile networks. However, a simple greedy forwarding by minimizing the distance to the destination location in each step cannot guarantee message delivery. Nodes usually have a limited transmission range and thus there are situations where no neighbor is closer to the destination than the node currently holding the message. Greedy algorithms cannot resolve such dead-end or local minimum situation. Therefore, recovery methods have been developed, the most prominent of which are based on planar graph routing, where the message is guided around the local minimum by traversing the edges of a planar subgraph of the network communication graph. Planar graph routing techniques can provide delivery guarantees under certain assumptions, which are explained in Section 3, and complement the efficient greedy forwarding. Altogether, greedy forwarding in combination with a recovery can be considered as state-of-the-art technique in geographic routing.

This chapter gives an overview of geographic routing starting with greedy algorithms and recovery strategies in Section 2 and Section 3, covering also theoretical results on their efficiency. Section 4 describes contention-based routing, a fully reactive variant of geographic routing, which does not require the neighborhood to be known in advance. In Section 5 algorithms and methods for geographic routing under realistic network models are reviewed. Further variants for multicasting and geocasting are described in Section 6. Section 7 describes methods for providing a location service that allows to look up the destination location. Section 8 gives examples for the application of geographic routing. Finally, Section 9 concludes the chapter.

1.1 Preliminaries

As the basic ideas of geographic routing use geometric criteria, we use the following concepts to describe wireless networks. An often-used and basic model for such networks is the so-called *unit disk graph* (UDG) which contains an edge $e = (u, v)$ between two nodes u and v, if $|uv| \leq 1$. In other words, assuming a restricted transmission range normalized to 1, the links between the nodes of a given node set form a unit disk graph. Unit disk graphs rely on the assumption that transmission ranges are uniform and imply that all links are bidirectional. We will see in Section 5 how this assumption can be weakened. Nodes within the transmission range are connected to the sender and called *neighbors*.

The network graph will be the basis to describe the routing problem. Routing starts at the source node s and aims at forwarding a packet to the target node t. The node currently holding the packet is called *forwarder*, possible next hop neighbors within its transmission range are called *candidates*.

2 Geographic Greedy Forwarding

The first approaches for geographic routing were developed in the 1980s for packet radio networks [84, 38] and wired networks [19]. These approaches describe local rules used by the forwarding node to select a neighbor as a next hop. As routing decisions are locally optimal, these approaches are termed *greedy forwarding*.

2.1 Greedy and Compass Routing

The first approaches for geographic routing were developed in the 1980s for packet radio networks [84, 38] and wired networks [19]. They are all greedy strategies, where the forwarding node makes locally optimal decisions to select next-hop neighbors based on the distance to the destination or on the so-called progress. In general, the next hop selection in greedy forwarding can be based on the following criteria (cf. Figure 1):

- Progress, i.e. the projection of the location of neighbor x on the s-t-line ($|x't| = \mathbf{st} \cdot \frac{\mathbf{st}}{|st|}$).
- Distance to the destination ($|xt|$) or advance, i.e. the distance gain towards the destination ($|yt| = |st| - |xt|$).
- Angular distance / angular separation ($\angle xst$).

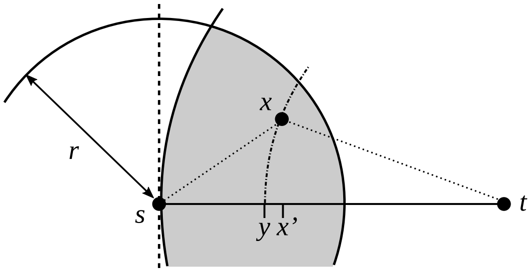


Figure 1: Progress and advance.

The notion of progress was used to define the MFR rule [84], which chooses the neighbor with the most forwarding progress within the transmission radius (see Figure 2). Similar to this rule, the *greedy method* in [19] selects the neighbor that minimizes the distance to the destination, which is equivalent to maximizing the advance. The angular criterion is used in Compass routing (CR)[48], where the neighbor is selected that minimizes the angle separation with respect to the destination.

Usually, the next hop in greedy forwarding is chosen among the neighbors with a positive progress (right of the dashed line in Figure 1) or with a positive advance (shaded area in Figure 1, also called *greedy area*). Selecting the next hop by the minimum distance or the maximum progress (MFR, greedy) gives an inherently loop-free forwarding rule independent of the unit disk graph assumption. Compass routing, which is based on the direction, is not loop-free [82].

Motivated by the observation that energy consumption can be reduced when using short links, required that the transmission range can be adjusted, the NFP (nearest with forwarding progress [38]) and NC (nearest closer [83]) criteria have been proposed. They select a neighbor which is closest to the forwarding node among all neighbors, but closer to the destination than the forwarding node itself, using distance or progress.

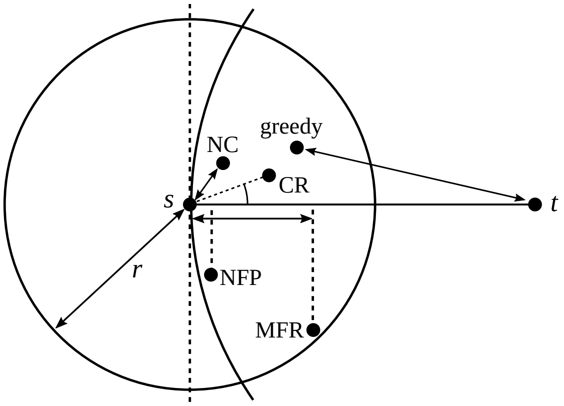


Figure 2: Variants of greedy forwarding.

2.2 Advanced Strategies

Greedy forwarding has one important drawback: it fails in local minimum situations where the forwarding node has no other neighbors closer to the destination. In some cases, a sophisticated strategy is necessary to recover from this situation; in other cases, a simple backward step is sufficient to be able to resume the greedy strategy successfully.

The GEDIR [82] method is such a greedy strategy with backward steps. Whenever a message has reached a local minimum, the packet is sent back to the previous hop, which applies the greedy rule again while excluding previously visited dead end nodes from the selection. This strategy is also loop-free.

Further improvements of the basic strategies Greedy, MFR and CR can be achieved if 2-hop information is available [82]. In this case, the forwarder selects a suitable neighbor out of the 2-hop neighborhood and forwards the packet to the direct neighbor that is connected to the selected node. Note, that 2-hop information has to be distributed, which requires a higher message overhead.

A greedy-based algorithm that goes beyond using 2-hop information is SPEED [34], which is designed to increase the relay speed. It uses an additional "backpressure" heuristic to avoid congested areas and void regions. The protocol relies on beaconing, extended by on-demand beacons for delay estimation and backpressure information. The forwarding works as follows: Nodes from the greedy area, whose relay speed is above a certain threshold, are selected probabilistically. The higher the relay speed the higher the probability to be selected. If no neighbor meets the relay speed requirement, the node drops the packet with a certain probability that depends on the failure ratio of packet forwarding to the neighbors. The necessary information to derive the failure ratio is gained from the neighbors by backpressure beacons, which are sent in case of congestion or in a local minimum situation. This method can alleviate local minima problems in case of small void regions, but it cannot guarantee delivery in general.

3 Planar Graph Routing and Recovery Strategies

Planar graph routing is a geographic routing strategy that is able to overcome the local minimum problem of greedy forwarding. Local minima exist at the border of *void regions*, where a node cannot find a neighbor closer to the destination than itself. Such nodes are also called *dead-end* nodes or *concave* nodes. Planar graph routing is a key concept for recovery from a local minimum situation. It is based on the idea that the network links form a communication graph, and a message can be routed along a sequence of faces in this graph. Routing along a face means that the nodes of a face pass the message along the incident edges by locally applying the *left-hand* or *right-hand* rule (see Figure 3). This rule is well-known from maze problems: One can find a way out of every simply connected maze when having the right-hand always in touch of the wall while walking. Applying the right-hand or left-hand rule to network graphs means to find a successor node in (counter-)clockwise order after the predecessor. This results in a *traversal* of a face of the communication graph. For a successful application of this rule, the underlying graph has to be planar.

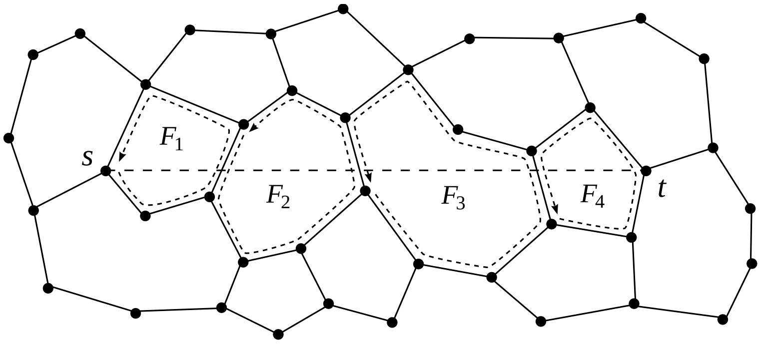


Figure 3: Planar graph routing: A path from s to t is found by sequential traversals of adjacent faces $F_1, ..., F_4$.

3.1 Face Routing

The first face routing strategies were proposed in 1999 by Kranakis et al. [48] and Bose et al. [10] under the names “Compass Routing II” and “Face-2”, respectively.

Compass Routing II traverses a sequence of adjacent faces until reaching the destination as shown in Figure 3. Each face is traversed completely in order to determine the edge that intersects the *s*-*t*-line and is closest to the target. Then the message is passed to an endpoint of this edge, where the face is changed and the traversal of the next face continues. Face-2 [10] also visits a sequence of faces, but it avoids the complete traversals and performs the face change before crossing the *s*-*t*-line, as depicted in Figure 4. On each face traversal, a node *u* checks whether the edge to the next node (u, u') intersects the *s*-*t*-line. If this is the case, then *u* changes the face and continues traversing the next face. A detailed description of face change rules can be found in [22]. Face routing has the advantage that it guarantees delivery on planar graphs while the nodes use only local position-based rules and do not need to keep state information.

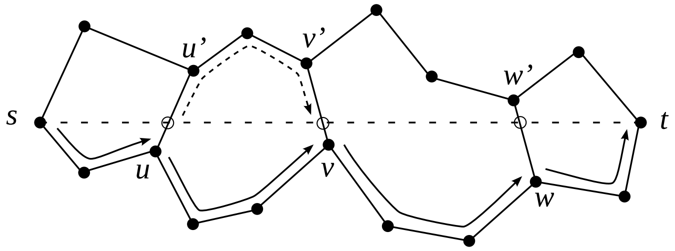


Figure 4: Face routing by FACE-2. A face change takes place at nodes *u*,*v*, and *w*

The planarity of the underlying network graph is required for assuring delivery guarantees, because crossing links as shown in Figures 5 and 6 can cause detours or routing loops. Therefore, an arbitrary unit disk graph has to be transformed into a planar graph first. This can be done locally by removing crossing edges using geometric criteria. The removal of edges however can increase the hop count, which makes face routing steps less efficient than greedy routing. Therefore, Bose et al. proposed the Greedy-Face-Greedy algorithm (GFG) [10], a combination of the efficient greedy forwarding and face routing on a planar subgraph to recover from local minima. A variant of this algorithm is known as GPSR [43].

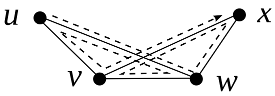


Figure 5: Crossing links causing a detour (starting from node *u*)

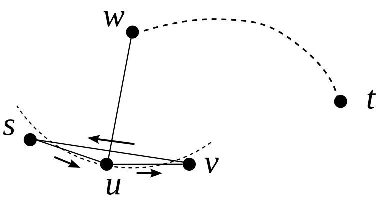


Figure 6: Crossing links causing a face routing failure

GFG and GPSR use greedy forwarding as long as possible. If greedy routing fails, a face traversal starts until the greedy strategy can be resumed. When starting recovery, the distance of the first node to the target d_r and the first edge e_r have to be stored in the packet header. If the first edge e_r is visited again for the second time, then the destination is not reachable and the packet is dropped. The distance d_r is used to check whether the next hop on the face traversal is closer to the destination than the node entering recovery mode. If such a node is found, greedy forwarding can be resumed instead of continuing the traversal until crossing the s-t-line (this is known as *sooner-back procedure* [14]). Pseudo-code for such a combined greedy and face routing algorithm is presented in the following. An example is shown in Figure 7.

A Combined Greedy/Face-Routing Algorithm

(GFG with sooner-back procedure [15])

Variables: previous hop p, current node u, target t, first edge in recovery mode e_r and distance to target d_r in rec. mode

```
if packet in greedy mode
  select next hop $v$ according to the greedy rule
  if no such neighbor exists
    select next hop $v$ in ccw. direction from $(u,t)$
    switch packet to recovery mode
    store current distance to the destination $d_r$
      and $e_r \leftarrow (u,v)$ in the packet header
  endif
else (packet is in recovery mode)
  if there is a neighbor $v$ with $||v-t|| < d_r$
    switch packet to greedy mode
  else
    select next hop $v$ in ccw. direction from $(u,p)$
    (using only nodes of a GG or RNG subgraph)
    if $(u,v)$ equals the first edge $e_r$ in recovery mode
      drop packet; return
    endif
  endif
endif
forward packet to $v$
```

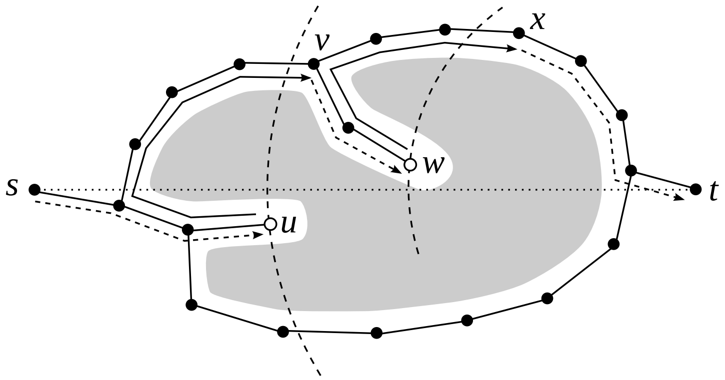


Figure 7: Combined greedy/face routing: After reaching local minimum *u* in greedy mode (dashed arrow), a face traversal is started (solid arrow) until a node *v* is found that is closer to the target than *u*.

When using such combined algorithms, the greedy part can be performed using all links of the unit disk graph, while face routing needs a local planar subgraph. We will see in the next section how a planar subgraph can be constructed.

3.2 Planarization

In their paper on face routing, Bose et al. [10] proposed a local planar subgraph construction based on the so-called Gabriel graph (GG) [27]. The Gabriel graph of a given point set contains an edge *uv* if Thales' circle on *uv*, i.e. the circle having *uv* as diameter, is empty. This circle is also called *Gabriel circle* over *uv* within this context. The Gabriel graph is known to be planar and connected.

This construction rule can be applied locally to a node's 1-hop neighborhood in order to extract a planar subgraph. The Gabriel graph construction and the so-called relative neighborhood graph (RNG) [86, 39] are the two most prominent local planarization schemes. Planarization using the GG criterion removes an edge *uv* if Thales' circle on *uv* contains another node *w*. Following the RNG criterion, an edge *uv* is eliminated, if the intersection of two circles with radius $|uv|$ centered at *u* and *v* contains another node *w* (see Figure 8). Applying the GG or RNG criterion to a unit disk graph yields a planar and connected graph, if the unit disk graph is connected.

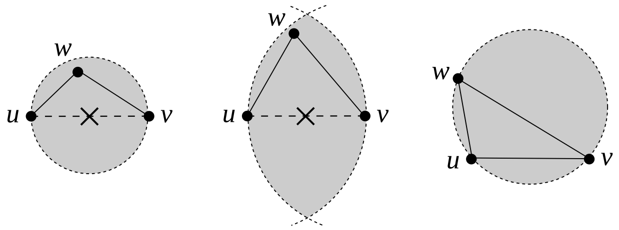


Figure 8: GG (left), RNG (middle) and Delaunay triangulation (right)

As planarization removes crossing edges it may induce detours due to missing edges. Therefore, planar subgraph constructions are desired that approximate the original graph as closely as possible. This property can be formally described by the so-called *spanning ratio* or *stretch factor*, which is defined by the maximum ratio of the shortest path length between two nodes u and v in the subgraph over the shortest path length between u and v in the original graph. A desired property is a constant stretch factor, as it guarantees a constant overhead for any path in the subgraph.

For Gabriel graph and RNG, however, the spanning ratio is unbound. Bose et al. [7] have proven length stretch factors of $\Theta(\sqrt{n})$ for the Gabriel graph and $\Theta(n)$ for the RNG, i.e. the detours induced by these subgraph constructions are not bounded by a constant. In terms of hop count, both GG and RNG have unbounded stretch factors as well.

To alleviate this problem, local construction schemes for the Delaunay triangulation were considered, which is known to have a constant spanning ratio. The Delaunay triangulation of a given point set contains all triangles whose circumcircle is empty (see Figure 8). In contrast to the GG or RNG, this criterion cannot be checked locally by using only 1-hop information. Therefore, variants of the Delaunay triangulation were considered, which can be constructed locally [29, 59, 60]. They are described in the following.

The Restricted Delaunay Graph (RDG) [29] is obtained by locally constructing Delaunay triangles, exchanging the local triangulations, and finally removing all edges uv if there is another node connected to u or v without having uv in its local triangulation. This scheme requires communication to obtain the desired subgraph, but provides a subgraph with constant stretch factor.

The Partial Delaunay Triangulation (PDT) [60] has been proposed in two variants, using either only 1-hop or 2-hop information. Both variants keep the Gabriel graph edges. A non-GG edge uv has at least one node in Gabriel circle over uv, and if there are two or more of such nodes left and right of uv within the circle, the edge is removed. If these nodes are located either only left or only right of uv within the circle, the maximum Delaunay circle among u, v and those 1-hop neighbors is considered. With 1-hop information, u keeps the edge uv if its transmission range covers the Delaunay circle (i.e. it is able to reach every other node within this circle) and the circle is empty. With 2-hop information u and v communicate and keep the edge if both cover the Delaunay circle and there is no other node within this circle.

The PDT scheme reduces the communication cost compared to the RDG, but it is unknown whether it can provide a constant factor spanner.

A localized reactive approach to planar subgraph construction has been presented in [21], which is called *Direct Planarization*. Direct planarization checks edges explicitly for possible edge intersections before they are used by the routing algorithm and decides in case of an intersection, which edge has to be removed. Two criteria were proposed, based on angle and Delaunay circles. Using the angle-based direct planarization (ADP), an edge uv is preferred over an intersecting edge wx, if either $\angle xuw$ or $\angle wvx$ is the maximum angle in the quadrilateral $uwvx$ (see Figure 9). The Delaunay-based direct planarization (DDP) favors an edge uv over wx, if x is not contained in the circumcircle of uvw.

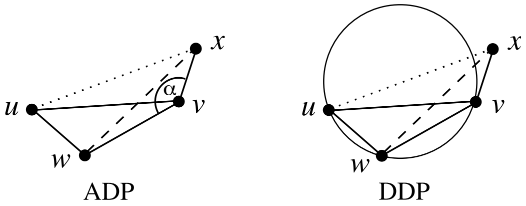


Figure 9: Angle-based and Delaunay-based direct planarization

Note that all the planarization criteria presented in this section rely on the unit disk graph assumption and bidirectional links. Problems of planar graph routing in non-unit disk graphs and possible solutions are described in Section 5.3.

3.3 Improved Planar Graph Routing

Though planar graph routing works stateless and uses local information only, it has two disadvantages: first, the construction of a planar subgraph removes edges and may increase the hop count. Second, a wrong choice of the traversal direction can lead to a long detour in the network.

The first problem is due to the planar subgraph construction. Planarization schemes such as the Gabriel subgraph construction remove long edges in favor of short ones, which results in an increased hop count when traversing a face of the resulting subgraph. This problem can be alleviated, if more information of the neighborhood is available. With 2-hop information, the forwarder can determine the local planar subgraph of its neighbors. Thus, instead of following the face traversal hop by hop, one can determine more next hops in advance and use *shortcuts* to the last predicted hop [14].

The Greedy Path Vector Face Routing (GPVFR) [57] is also based on the idea to exploit multi-hop information in order to broaden the horizon for forwarding decisions. GPVFR distributes information among the nodes of the same face. If greedy forwarding fails, a node might know another node on the same face being closer to the destination. Then this node is chosen as an intermediate target, which can be reached by face routing, even if the complete information about the current face is not given.

Another technique to reduce the detour on a planar subgraph is to use the nodes of a connected dominating set [14, 1]. In a dominating set, each node is either part of the set, or adjacent to such node. A connected dominating set can be constructed locally using only position information of the 1-hop neighborhood. Then, the planar subgraph construction and the face traversal is performed on the internal nodes of the dominating set only, which usually leads to a better subgraph than obtained by planarization of the original network graph. It was shown by Datta et al. [14] that using shortcuts and internal nodes of the dominating set can effectively reduce the hop count of face routing or GFG. Especially the use of internal nodes contributes significantly to a hop count reduction while requiring only 1-hop information for determining the internal node status.

Geographic clustering can also reduce the complexity of the underlying topology. Nodes can be grouped into clusters that are defined by geographic areas. In [70] the nodes are mapped onto a grid of quadratic cluster areas. This grid structure provides the planarity property implicitly and can be used for a virtual routing along grid cells. In [20] the unit disk graph is first planarized using the Gabriel graph criterion. Afterwards a clustering into hexagonal areas is applied. These clusters are the basis for a cluster graph, which contains an edge between two clusters, if nodes within the respective hexagonal cells are connected. The cluster graph is planar and provides the basis for face routing. The advantage of geographic clustering is that a face routing algorithm has the freedom of choice among the nodes within a cluster. It is also more robust in mobile scenarios, because the resulting graph does not change until a cluster cell changes is entered by a first node or left by the last node, i.e. it changes its status.

The second problem of planar graph routing is to find the right traversal direction when starting a face traversal. As an example, if node u in Figure 10 chooses v as next hop, the traversal of F_3 during face routing leads through w along the complete outer boundary until reaching the s-t-line again. A traversal of F_3 in the opposite direction is here the better choice as it crosses the s-t-line after 5 hops. Choosing the optimal direction is not possible with only local information, but the detour can be bounded by restricting the search area. Adaptive Face Routing (AFR) proposed by Kuhn et al. [50] is a face routing variant that restricts the search area by a bounding ellipse during face traversals (see Figure 10): Whenever the message reaches the ellipse on the traversal path, it is sent back into the opposite direction. The face traversal is finished when the ellipse is reached for the second time, or if the traversal has reached the start point. After the traversal, the message is sent to the node closest to the destination on the current face. This node changes the face and continues the traversal of the next face (white nodes in Figure 10). The original face routing, by contrast, performs the face change when crossing the s-t-line.

The bounding ellipse has s and t as foci and the initial size of the major axis depends on the implementation. In [52] Kuhn et al. recommend to use an initial size of the major axis of $1.2|st|$ and an enlargement factor of $\sqrt{2}$. If there is no path to the target within the bounding ellipse, the size of the ellipse is doubled and the source starts face routing again.

AFR can be used on its own or as as a recovery strategy in combination with greedy forwarding. This combination is called Greedy Other Adaptive Face Routing (GOAFR) [52]. A further improvement of this algorithm, called GOAFR+ is presented in [49]. It employs method other than the bounding ellipse to restrict the search area, namely a circle centered at the destination, which initially contains the source node and is gradually reduced when the packet approaches the target. Furthermore, GOAFR+ uses an early fallback strategy to leave the recovery mode sooner to resume greedy forwarding. In contrast to the sooner back procedure, which leaves greedy mode as soon as the distance to the destination is decreased since entering face routing mode, GOAFR+ uses the following criterion: It maintains two counters for the nodes on the traversal that are closer and for those that are not closer to the destination than the start node of the traversal. If the number of nodes closer to the destination exceeds the rest of the nodes on the traversal by a certain factor, the traversal is stopped and the message is passed to the node that is closest to the destination among the visited nodes and greedy forwarding is resumed. The use of the two counters avoids multiple traversals of nodes that were already participating in a traversal before. Through this strategy the algorithm retains its asymptotic efficiency. An overview of AFR and its variants is given in [52].

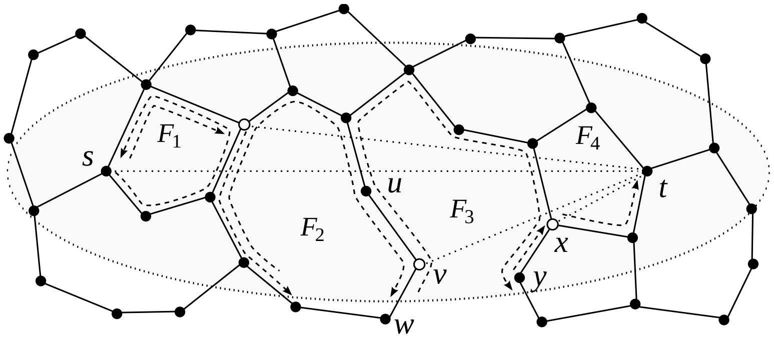


Figure 10: Adaptive Face Routing (AFR) [52]. Faces are traversed completely while the search area is restricted by a bounding ellipse.

3.4 Recovery beyond Planarization

Recovery from local minima does not necessarily require creating a planar subgraph prior to routing. One approach is to identify possible local minima beforehand, such that a path around a void region is already established when a packet in greedy mode arrives at a dead-end node. For this purpose Fang et al. [16] propose the Bound-

Hole algorithm, that identifies so-called stuck nodes by applying geometric rules locally. Stuck nodes are located on the border of a void region. They are not dead-end nodes in general, but for some specific target locations they are local minima. The void region can be identified then by communication among the stuck nodes. The BoundHole algorithm can be used in conjunction with a combined greedy and face routing algorithm. When reaching a local minimum, the face routing part can use the pre-computed information about the border of a void region and immediately select the shorter path around this region. This yields shorter paths at the expense of an additional communication overhead for the communication among stuck nodes.

A similar idea led to the NEAR algorithm [2] that assigns virtual coordinates to nodes in a dead-end region. These coordinates are called elevation values and based on the angle between a node and its neighbors. Elevated nodes are avoided by the routing algorithm. A typical situation where high elevation values are assigned is a "peninsula" of nodes which extends into a void region. The nodes in this dead-end region are elevated successively such that the dead-end region is predictable before entering it. This mechanism is complemented by a void identification algorithm that is started by the nodes at the border of a void region. Thus the perimeter information is available beforehand and can be used to find shorter routes around void region.

Another approach for identifying the boundary of void regions is presented in [63] as part of the Greedy Anti-void Routing (GAR) routing algorithm. Here, the idea for the boundary traversal is to roll a ball whose diameter is the transmission range along the nodes on the boundary of a void region. The next node v that is hit by the ball satisfies that the edge from the previous node u is not intersected by another edge having one endpoint neither connected to u nor v. Thus, it is ensured that no important edge on the traversal is missed. The algorithm establishes pointers along the traversal direction which are used to circumvent void regions.

The described techniques are based on geometric criteria to identify possible dead-end nodes. There are further recovery techniques that do not require a unit disk graph. One of the underlying concept is an identification of crossing links or paths around void regions by graph traversals. These approaches are described in Section 5.3 on recovery in non-unit disk graphs.

3.5 Delivery Guarantees and Asymptotic Efficiency

The prominent geographic routing algorithms GFG, GPSR and GOAFR+ have been subject to simulative studies as well as to theoretical considerations. All three algorithms guarantee delivery on GG or RNG subgraphs, which can be constructed locally under the unit disk graph assumption. GFG provides this delivery guarantee even on any planar graph. Frey and Stojmenovic showed that the delivery guarantees on GG and RNG are due to a structural graph property of these graphs [22]. In GG and RNG there is always an edge intersecting the s-t-line having one endpoint closer to the target t than the source s, and this implies that a face traversal always finds a node closer to the target.

The performance of the aforementioned geographic routing algorithms depends mainly on the face routing part. Like in all combined greedy/face routing algorithms, the face routing part is needed to find paths around void regions and the resulting detours significantly influence the performance. Consider a face traversal starting at node u in Figure 10 as an example: Turning left at this point is obviously the shorter path to the target, while turning right leads along the boundary of the network which can result in an arbitrarily long detour. An efficient algorithm finds a path whose length is close to the optimal path length. Therefore, the efficiency is described with respect to the shortest path length or, more generally, to the optimal cost. This is also termed a *competitive* measure.

In situations as shown in Figure 10, where a traversal can lead along the network boundary, GFG and GPSR produce worst-case paths that cannot be bounded by a function of the shortest path length. A bounded detour can be achieved by the bounding ellipse technique, which was introduced in Adaptive Face Routing [50] and used for GOAFR and its variants. Kuhn et al. showed that AFR finds a path of cost $O(c^2)$ in the worst case, where c is the cost of the optimal path. The quadratic bound is obtained because the detour is limited by the area of the bounding ellipse and doubling the ellipse does not affect the asymptotic complexity. For this cost bound it is required that the minimum distance between any two nodes is bounded by a constant, which is called $\Omega(1)$-model. The GOAFR+ algorithm, which in addition performs greedy routing, has the same asymptotic efficiency. Kuhn et al. also prove that there is a quadratic lower bound on the cost. Therefore, AFR and GOAFR+ are asymptotically optimal.

The lower bound construction from [50] is depicted in Figure 11. It consists of $2k$ nodes on a ring that are just able to reach each other. Every second node on this ring is attached to a chain of $\Theta(k)$ nodes pointing towards the center. Only one of these chains leads to the target. In this graph any deterministic or randomized geographic routing algorithm needs to explore at least half of the dead-end chains in the (expected) worst case, before finding the target. This results in a path cost of $O(k^2)$ while the shortest path cost is $O(k)$.

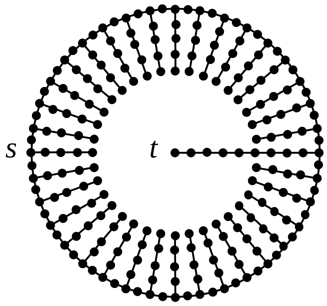


Figure 11: Lower bound construction for geographic routing algorithms [50]

This lower bound holds for any randomized or deterministic geographic routing algorithm. A similar lower bound holds for greedy algorithms only. Greedy algorithms cannot guarantee delivery in the worst case, but if there is a greedy path, it is not even close to the optimal path in the worst case. Considering the graph in Figure 12, a greedy choice based on distance minimization leads from s to node v that is closer to the target. The path from v to t brings the message closer to the target in each step, but its length is quadratic in the length of the optimal path, which leads from s through u to t. Thus, for distance-based greedy forwarding, there is a quadratic lower bound with respect to the shortest path length [29].

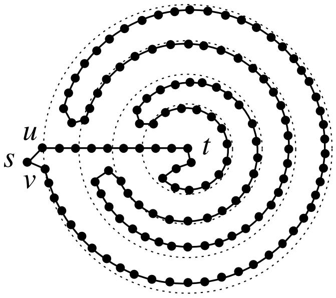


Figure 12: Lower bound for greedy forwarding [29]

Considering the hop count metric, the quadratic lower bound for geographic routing implies that a quadratic number of messages is needed. This bound on the message complexity can be reached by a flooding algorithm. Actually, flooding visits all nodes in the network, but it can be limited by a doubling technique that repeats restricted flooding while doubling the hop limit. This technique is also known as expanding ring search [41] and has a quadratic message complexity. The quadratic lower bound, which is defined in terms of the shortest path, suggests that geographic routing strategies have the same worst case message complexity than flooding algorithms, while flooding algorithms are obviously faster.

In fact, geographic routing is not as inefficient in the worst case as it seems. We have seen that the difficulty of the geographic routing problem depends on the void regions which are local minima to a greedy strategy and require a traversal. Following the rationale in [5], one can show that in the worst case the traversal of void regions is unavoidable. Thus, there is a lower bound on hop count and message complexity of $\Omega(d+p)$ for geographic single-path strategies, where d is the shortest path length and p the perimeter of void regions. This analysis uses an abstraction from the geometric issues connected with the problem of void identification and graph planarization: By a geographic clustering technique [70], a unit disk graph can be represented by a grid with usable and defective regions. This way the geographic routing problem on unit disk graphs can be transformed to the problem of routing on a defective grid with only local information. It inherently provides a minimum distance property as defined in the $\Omega(1)$-model.

The lower bound graph in Figure 11 states a special case of the generalized lower bound of $\Omega(d+p)$ with $p=\Theta(d^2)$, i.e. having a total perimeter length that is quadratic in the shortest path length. The fact that any single-message strategy has to examine the complete perimeter in the worst case leads to the quadratic bound for hop count and message complexity in this case. The question whether geographic single-path strategies are as inefficient as flooding in terms of messages can also be answered by considering the perimeter length. While geographic single-path strategies can find paths of length $O(d+p)$ using the same number of messages, expanding ring search (as a flooding or multi-path strategy) can route a packet in $O(d)$ steps, but uses always $O(d^2)$ messages, regardless of the perimeter length. As a result, geographic routing is efficient, if void regions are small in comparison to the shortest path length. It also raises the question whether there is an algorithm that is as fast as flooding, but uses less messages. This is answered in [71] by a multi-path strategy that approaches the lower bound of $\Omega(d+p)$ on the message complexity up to a poly-logarithmic factor, while preserving the asymptotic optimal time bound of $O(d)$.

Other competitive algorithms in this context have been presented by Bose and Morin [9, 8] for routing in triangulations and planar graphs with certain properties. However, the considered graphs have no local minima, which does not comply with the unit disk graph model.

4 Contention-based Routing

The geographic routing algorithms described so far require the locations of 1-hop neighbors to be known as stated by the second assumption for geographic routing in the Introduction. This knowledge can be acquired by a regular exchange of beacon messages containing the own position information. However, the knowledge of the neighbors' positions is not required for forwarding a message greedily. Indeed, without knowing the neighborhood, the forwarding node cannot select the next hop explicitly, but the next hop can select itself in a contention with other neighbors. This kind of routing is called *contention-based* or *beaconless* routing, as a beacon exchange is not required.

4.1 Contention-based Greedy Forwarding

The first contention-based georouting algorithms were proposed in 2003 by independent groups under the names Beacon-less Routing (BLR) [35], Contention-based Forwarding (CBF) [26], and Implicit Geographic Forwarding (IGF) [6]. They all follow the same principle: First, the forwarder broadcasts the message to its (unknown) neighbors. Some of the neighbors, which are located in the *forwarding area*, an area closer to the destination and where all nodes can overhear each other, are the *candidates* for the next hop and contend for the message (see Figure 13). They set a set a timer in accordance to their distance to the destination. More specifically, the timer is determined by a *delay function* depending on the advance, such that the candidate closest to the destination has the shortest timeout. Such a delay function is defined as follows, where a is the advance of the candidate, r the transmission range, and $t_{\max}$ the maximum timeout, which defines the length of the contention period.

$$t(a) = \tfrac{a}{r} \cdot t_{\max}$$

Once the first timer expires, the respective candidate re-transmits the message. The other candidates overhear the re-transmission and cancel their scheduled transmissions. The re-transmission also serves as a passive acknowledgement to the forwarder.

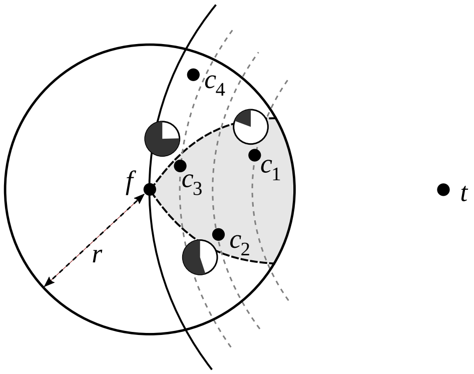


Figure 13: Contention-based routing. Only the candidates in the forwarding area (shaded) participate in the contention. Candidate c_1 has the shortest timeout and takes over the message.

The choice of the forwarding area is implementation-dependent, and there are a few possibilities (see [35, 87] such as a 60°-sector directed towards the target, a circular region, or the Reuleaux triangle, which is shown in Figure 13. The only prerequisite is that all nodes within this area have to be able to overhear each other. Otherwise, two candidates could re-transmit the packet without knowing each other, which leads to an undesired packet duplication. Another problem pointed out in [87] occurs, if two candidates c_1 and c_2 start their re-transmission almost simultaneously. Then, these two nodes initiate two contention-rounds in parallel. The new candidates in the intersection of the transmission range of c_1 and c_2 receive the first transmission by c_1 and also overhear the second transmission by c_2, which leads to the misinterpretation that c_2 won the contention and became the next hop. If in this situation there is no other candidate that could hear only one transmission, the forwarding stops and the packet is dropped. This can be solved by storing a hop counter in each packet, so that a candidate cancels its scheduled transmission only if another node re-transmitted the packet before and the hop counters are different.

This timer-based or contention-based selection of the candidate enables a completely reactive greedy forwarding without any message overhead. However, the contention costs a certain delay and the candidates are selected from a rather restricted area. It is possible to use the complete greedy area, i.e. all neighbors closer to the destination than the forwarder participate in the contention process. This has a positive effect on the performance [12], but it requires that the forwarder actively selects the candidate, which needs additional control messages. This scheme works as follows: The forwarder broadcasts a *request to send* (RTS) to its neighbors. The neighbors being closer to the destination set a timer in accordance to their distance to the destination. The first candidate whose timer expires replies with a *clear to send* (CTS). Then the forwarder sends the data packet immediately. The other candidates overhear the CTS by the candidate or the data transmission by the forwarder and cancel their schedules replies. Finally, the forwarder sends the packet to the selected candidate by unicast.

The previously described scheme implicitly assumes a continuous time model. Practically, one cannot require a fine-grained time resolution such that no nodes answer simultaneously and cause a packet collision. Geographic Random Forwarding (GeRaF) [89] is an approach that solves the problem of colliding control packets. GeRaF uses the RTS/CTS scheme and a forwarding area that includes all neighbors closer to the destination than the forwarder. The forwarding area is divided into zones and the candidates within these zones contend for being the forwarder in the next round. If two candidates from the same zone cause a collision while sending their CTS, they will retransmit it with probability $1/2$ in the next round. This probabilistic drop out reduces the probability of collisions step by step because colliding nodes from the last round survive with probability $1/2$.

In networks where the nodes can adjust their their transmission power, energy can be saved by adapting the transmission range to a suitable relay node. In beaconless protocols, however, the problem is that the relay nodes are not known in advance and suitable candidates have to be found by broadcasting requests at increasing transmission ranges. As this probing itself consumes energy, the question is how the transmission range should be increased. Moreover, if a candidate is found, is it worth to increase the transmission range in order to search for a better candidate? Galluccio et al. [28] address this problem and propose a beaconless routing protocol that discovers candidates by such probing strategy. In the first round of the proposed algorithm, candidates are discovered by sending an RTS at an initial transmission range, and the best neighbor replies. Then the transmission range is increased, but re-transmitting an RTS at a higher transmission power is connected with some cost. The optimal point for stopping a further range increase is found when the benefit of finding a better neighbor is smaller than the cost of the range increase. The benefit here is a progress factor which is essentially the same as the ratio of the transmission cost over the progress achieved. The concept of cost-over-progress will be explained in Section 5.1 in the context of realistic transmission models.

4.2 Reactive Recovery Strategies

Contention-based greedy forwarding suffers from the same local minimum problem as described for conventional greedy algorithms. In a contention-based or beaconless routing algorithm, the recovery cannot rely on known neighborhood information. A straightforward solution to reactive recovery is the request-response approach of BLR [36]. Whenever the forwarder finds no candidate, it request all neighbors to send their position. With this 1-hop knowledge, a local planar subgraph construction and face routing as described in Section 3 is applied. This strategy can be seen as reactive beaconing as it involves the complete neighborhood in exchanging position information.

A reactive scheme for face routing, which guarantees delivery and does not require communication with all neighboring nodes, is described in [42]. The main problem is that face routing requires an underlying planar subgraph and planarization requires some knowledge of the neighborhood. According to the two tasks of planarization and selecting the next hop edge, two strategies have been proposed: Beaconless Forwarder Planarization (BFP) performs the reactive planarization first, such that face routing can continue; Angular Relaying selects the next edge on a face traversal first and checks then, whether the selected node is part of a planar subgraph.

BFP is a generalization of the earlier proposed GDBF protocol [11] and can be used with the Gabriel graph and the relative neighborhood criterion. BFP is proposed together with a modified subgraph construction which is similar to the Gabriel graph but allows to bound the number of messages, which is not possible when using the Gabriel graph criterion. In contrast, the request-response approach of BLR cannot provide this bound on the number of messages as it involves communication with all neighbors. BFP uses a delay function of the distance to the forwarder, i.e. the neighbor closest to the forwarder may reply first:

$$t(d) = \frac{d}{r} \cdot t_{\max}$$

If a neighbor v overhears an earlier reply by another neighbor w, it checks whether w is located within the Gabriel circle over fv. If this is the case, the edge fv is not part of a Gabriel subgraph and thus v becomes a *hidden node* and cancels its reply. Now the problem is that v itself could violate the Gabriel graph condition for a neighbor that responds later. Thus, the hidden nodes are given the opportunity to protest against replies in a second phase. An example is shown in Figure 14. Finally, the forwarder obtains the neighborhood of a local planar subgraph and can apply any planar graph routing algorithm.

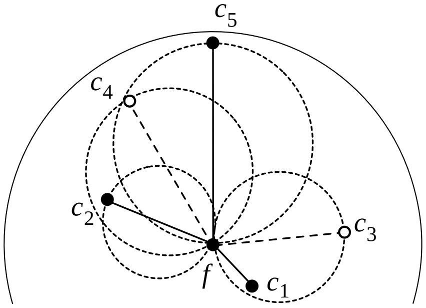


Figure 14: Beaconless Forwarder Planarization (BFP) using the Gabriel graph criterion. The candidates reply in the order c_1, c_2, c_5; c_3 and c_4 are hidden nodes. In a second phase c_4 protests against c_5.

Angular Relaying finds the next neighbor in counter-clockwise order and checks then if the connecting edge fulfills the Gabriel graph condition. It follows a *select-and-protest* principle: If the first candidate in counter-clockwise order is selected, then another candidate with a larger delay, which has not replied at this point, may lie within the Gabriel circle over the selected edge. This candidate has to be given the chance to correct the initial decision. Thus, this candidate sends a protest message, that is received by the forwarder and the previously selected candidate as well. Now the protesting node becomes implicitly selected. If there is no other node violating the Gabriel condition for the new edge, the contention ends and the currently selected candidate will receive the data packet.

Angular Relaying as depicted in Figure 15 uses a delay function that depends on the angle θ between the last hop, the forwarder and the candidate:

$$t(d,\theta) = \frac{\theta}{2\pi} \cdot t_{\max}$$

With this delay function, the area of possibly protesting nodes is half of the Gabriel circle over the currently selected edge. In this area, the nodes have a longer timeout than the currently selected node and may not reply before this selection. When using other delay functions, the area of possible protests can be reduced. Further details can be found in [42].

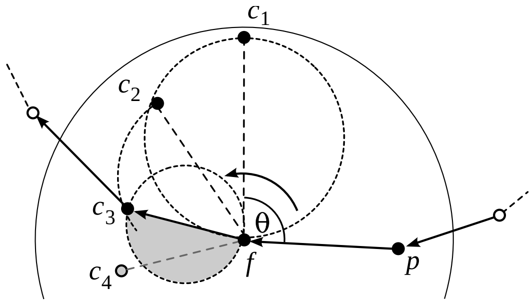


Figure 15: Angular Relaying

5 Georouting under Realistic Network Models

In practice the transmission radius is not uniform as assumed in the unit disk graph model. Usually one can observe stable links near to the transmitter and no connection beyond a certain distance to the transmitter. In between there is a transitional region with volatile connections [90]. In order to transform this observation into an abstract model, the following two approaches have been proposed:

From a more graph-theoretic perspective, weakening the unit disk graph assumption has led to the definition of *Quasi Unit Disk Graphs* (QUDG) [3, 51]. A Quasi Unit Disk Graph contains all edges shorter than r and no edges longer than a cutoff distance R (see Figure 16). Nodes in between, i.e. at a distance between r and R, may or may not communicate directly, which corresponds to the transitional region between stable short-range connections and the cutoff distance.

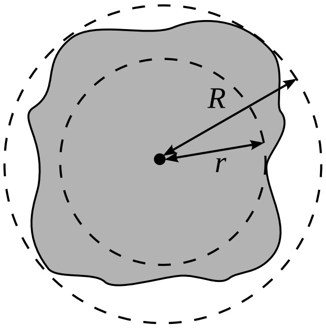


Figure 16: Quasi Unit Disk Graph

From a physical-layer perspective, the stability of communication links decreases as the distance to the transmitter increases. Based on the lognormal shadowing or log-distance path loss model, this can be modeled by a function that returns the reception probability for a given distance [68, 54] (see Figure 17).

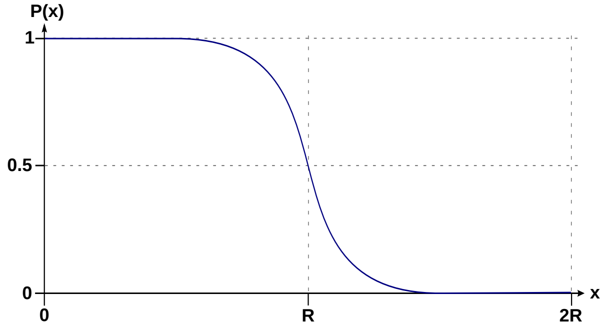


Figure 17: Reception probability versus distance

Both models were used to investigate geographic routing strategies under more realistic assumptions. We will see in the remainder of this Section that in greedy forwarding the consideration of reception probability has led to new local forwarding rules, while in planar graph routing and recovery the Quasi Unit Disk Graph model has been studied.

5.1 Greedy Forwarding with Realistic Physical Layer

Under realistic physical layer considerations, the pure greedy forwarding, which selects the neighbor with the maximum advance in each step, does not yield optimal results, because it is designed with respect to the hop count metric and does not take lossy links into account. If a node in the transitional region with low link quality is selected as a next hop, it might take a few transmission attempts to successfully forward a packet, whereas another node with a smaller advance could be reached on the first try. Hence, the metric for doing such forwarding decisions should include the cost for re-transmissions [75]. The cost for re-transmissions can be estimated by the number of transmissions needed to send a data packet over a specific link. By comparing this to the progress a link can provide, one can find a balance between costly retransmissions over long distance links and the increased hop count when using short links.

The expected number of re-transmissions can be derived from the packet reception rate (PRR). The packet reception rate is the ratio of successfully received packets over the total number of transmitted packets over a specific time period. If this data is recorded by a node, it can serve as a reception probability p for future transmissions. The reciprocal $1/p$ describes the expected number of transmissions needed for a successful reception, if acknowledgements are not taken into account.

Seada et al. [73] propose to use the product of the packet reception rate and the advance towards the destination as a metric for forwarding decisions. As a low PRR causes a high number of re-transmission attempts, this metric penalizes lossy links. Assumed that the packet reception rate for communicating with the neighbors is known, the forwarder chooses a neighbor based on this metric. In addition the authors propose different blacklisting strategies to filter out the farthest neighbors or the neighbors with the worst reception rate.

A general metric for dealing with a realistic physical layer model is the cost-over-progress ratio presented by Kuruvila et al. [54, 55]. Minimizing the cost-over-progress ratio means to find a next hop neighbor that requires the minimal cost for transmission per progress towards the destination. The PRR × distance metric in [73] also follows this concept, as it is essentially a ratio of the advance over the expected number of transmissions, and maximizing this ratio is equivalent to minimizing the cost over progress. Note, that cost can be an arbitrary measure and include the number of re-transmission attempts or energy-related metrics. Likewise, progress can be defined in terms of distance, by which the packet advances, or by the projected distance as in the MFR rule for greedy routing.

Kuruvila et al. use the cost-over-progress concept in their expected progress routing (EPR) [54] algorithm. Progress is here defined as the advance towards the destination. As a cost metric they use an expected hop count (EHC), which is based on the reception probability p and defined as the expected number of transmissions including data and acknowledgements. It is calculated by $1/p^2 + 1/p$, which includes the retransmissions due to lost acknowledgements. EPR selects the neighbor that minimizes EHC over advance.

Sometimes it can be more efficient if the selected neighbor is not reached directly, but through an additional intermediate hop. Therefore, an iterative version of this algorithm has been proposed, which works as follows: The forwarder f first selects a neighbor v that minimizes the EHC over advance. Afterwards it tries to find an intermediate node u for which $\text{EHC}(f,u)+\text{EHC}(u,v)$ is the minimum of all neighbors and smaller than $\text{EHC}(f,v)$. If such neighbor exists, it will be selected as a next hop (see Figure 18). An overview of the cost-over-progress framework can be found in [81].

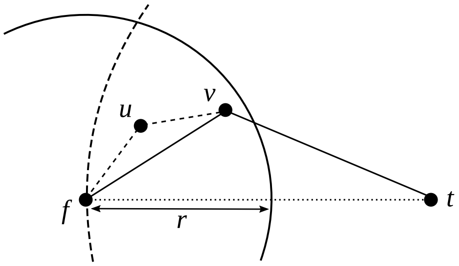


Figure 18: Iterative expected progress routing.

5.2 Localization errors and mobility

Geographic routing relies on precise position information, which is in practice not always available. Inaccurate positioning devices or node mobility can lead to wrong or out-dated position information. Especially in mobile scenarios the approaches that rely on beaconing to announce position information bear the problem of out-dated information. Contention-based strategies are advantageous in those scenarios as position information is acquired on demand. But also reactive methods are confronted with problems in highly mobile scenarios. A node moving out of range during a transmission can cause a short-term asymmetric link. This can lead to packet loss (in the unacknowledged case) or to packet duplication as the sender is not aware that the packet was received and is already being forwarded. This problem is not specific to geographic routing. However, if the localization method or the positioning device is unreliable, one has to deal with localization errors even in static scenarios.

While greedy forwarding is almost not affected by moderate localization errors (<25%) [33], recovery strategies such as face routing rely on geometric criteria, which require precise position information about the neighborhood. The effect of such errors on planarization and face routing has been studied by Seada et al. [72]. Localization errors cause two main problems: First, an incorrect position information broadcasted to the neighbors can lead to a network disconnection during planarization when using local criteria such as the Grabriel graph condition (see Figure 19). A network disconnection after applying the GG or RNG planarization can be prevented by requiring a mutual witness: An edge is only removed, if both endpoints are connected to the node violating the GG or RNG condition. The second problem occurs, if a crossing edge is not removed during planarization, because of a wrong estimated position. As a result, face routing will operate on a non-planar graph.

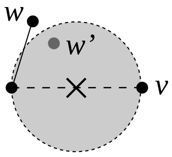


Figure 19: Localization error causing disconnection after planarization

The effect of mobility on geographic routing is investigated by Son et al. in [74]. They identify two problems: lost links and loop problems. Lost links occur mainly in greedy forwarding if nodes close to the border of the transmission range are used. This is specific to the plain greedy method, other strategies based on cost-over-progress usually select candidates closer to the forwarder, which alleviates this problem. For mobility-related problems, Son et al. propose a mobility prediction strategy. The nodes extrapolate the new position of a neighbor from previous beacon information, such that they can estimate future locations of their neighbors.

5.3 Recovery in Non-Unit-Disk-Graphs

In networks that do not obey the unit disk graph assumption the planarization schemes described in Section 3.2 do not work correctly and can cause a network disconnection as shown in Figure 20.

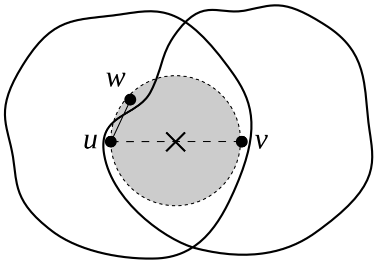


Figure 20: Disconnection after planarization due to irregular transmission ranges

Under a weaker condition than the unit disk connectivity the planarization and a subsequent face routing is possible. Such condition was introduced by Barrière et al. [3] and later generalized to the *Quasi Unit Disk Graphs* (QUDG) by Kuhn et al. [51]. In a Quasi Unit Disk Graph with minimum and maximum transmission ranges r and R, links of length smaller than r are guaranteed and no links longer than R exist. If all links are bidirectional and the ratio R/r is at most $\sqrt{2}$, a planar graph can be constructed by using only local information [3]. This is possible because any node within the Gabriel circle over a link uv is either connected to u or v under this condition. Therefore, nodes u and v can agree by communication on whether the Gabriel condition for the edge uv is fulfilled or the edge has to be removed. If the edge uv is removed because of a node w inside the Gabriel circle over uv, a *virtual edge* between u and w is introduced, if w is connected to v, but not to u. Analogously, a virtual edge vw is created, if vw does not exist in the original graph. The Gabriel graph condition is applied to virtual edges as well, such that the removal of a virtual edge requires the introduction of a new virtual edge. The resulting subgraph extended by virtual edges is planar and connected, such that face routing can be applied. Routing over virtual edges is then performed by communication of the endpoints, which are always connected by a path in the original graph. The drawback of this method is that this requires routing between the endpoints over multiple hops in the original graph. The length of this routing path can be bounded if there is a minimum distance between any two nodes; otherwise, it is unbounded (see the discussion in [3]). The route length bound can be preserved without the minimum distance assumption by using a backbone construction based on clustering as described in [53].

Instead of introducing virtual edges, one can define virtual nodes for edge intersections. In QUDGs with $R/r \leq \sqrt{2}$ these intersections can be detected locally and thus the endpoints of the intersecting edge can negotiate the routing steps by direct communication [52]. Nesterenko et al. [67] propose such a strategy, where face traversals are performed on the network graph without prior planarization using a concept equivalent to the aforementioned virtual nodes. If the face traversal reaches a crossing edge, the next edge is determined after communication by the endpoints (Figure 21). For this algorithm, a geometric property such as the QUDG with $R/r \leq \sqrt{2}$ is not required. However, as the endpoints need to communicate, the authors require the network to satisfy a topological property called intersection semi-closure, which states that the endpoints of intersecting edges are connected by paths of restricted length. VOID routing can be combined with greedy forwarding and guarantees delivery in networks that obey the semi-closure property.

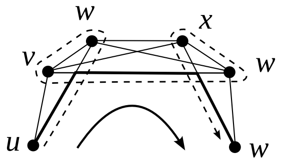


Figure 21: The VOID routing principle [67]

Irregular transmission ranges in general result in Quasi Unit Disk Graphs where the ratio R/r exceeds $\sqrt{2}$. The main problems in such graphs are planarization errors as described in the previous section, and incorrect face changes [45]. The correctness of various face routing algorithms in those graphs depends on details regarding the place of face change and chosen traversal deriction. A detailed analysis on this issue can be found in [22]. The bottom line is that GFG can guarantee delivery on any planar graph, and the remaining problem in networks under irregular transmission ranges is to remove crossing links.

Kim et al. [44] propose the Cross Link Detection Protocol (CLDP) that detects and removes crossing links through repeated face traversals. This is achieved through probing messages that contain the information about the end points of a link. While this message is sent along a face, each node checks whether the next link on the traversal intersects the edge specified in the message. This link is marked non-routable unless the probing message has traversed this link twice in opposite directions – in this case a removal would cause a network partition. After this preprocessing the planar graph routing continues on the resulting network graph. As this method incurs a significant message overhead, the same authors propose a strategy that does not start the cross link detection pro-actively, but on-demand whenever the routing protocol selects a link [46].

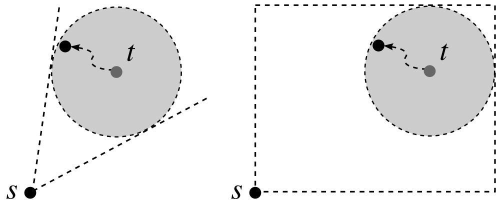
Similar in spirit to the cross link detection protocol is the face tracing method by Zhang et al. [88]. The preprocessing comprises the following steps: In a first step, clusters of nodes are built with the clusterhead being adjacent to all cluster members. Then a cluster graph is set up that represents the interconnection of the clusters. In the cluster graph the longest edge of each triangle is removed. Then the faces of the cluster graph are identified by sending inquiry messages along the incident edges. Each face is given an ID, which the incident nodes remember. Routing on this graph is done by traversing the faces of the cluster graph (*face tracing*). The IDs of traversed faces are stored in the message, and the order of the traversed faces is selected by a depth-first search. This guarantees that all parts of the network can be reached.

A further strategy for routing in non-unit disk graphs is GDSTR [56], a protocol that is based on a distributed spanning tree construction. This spanning tree contains in each node the information about the area that is covered by the subtree. More precisely, a node stores the convex hull of all node locations in the subtree and also the convex hull information of its descendants. For a recovery from a local minimum, a message is routed upwards in the tree until a node is found, whose convex hull contains the destination. From this point, the message is routed downwards in the tree until the destination is found.

Recovery strategies for arbitrary non-unit disk graphs such as CLDP and GDSTR need multi-hop communication to identify crossing links or to gather the necessary information for successful recovery, i.e. these protocols are not localized. On the other hand, localized protocols such as GFG, GPSR and GOAFR cannot guarantee delivery in arbitrary non-unit disk graphs.

6 Geocasting and Multipath Strategies

Multi-path strategies send duplicates of a message along different paths to the destination in order to increase the delivery rate and the resilience to routing errors. These strategies can overcome routing problems that are due to mobility and out-dated position information. Especially out-dated location information about the destination node can lead to delivery failures under mobility. DREAM [4] tries to overcome this problem by letting each node forward a message to all neighbors that are lying within a cone including the expected target area (see Figure 22 left). The target area is a circle around the last known position of the target having a radius that represents the maximum possible movement since the last position update. This strategy is also termed *restricted directional flooding*. In a similar way, LAR [47] restricts the flooded area to a rectangle including the expected target area (see Figure 22 right).



(a) DREAM (b) LAR

Figure 22: Multi-path routing by DREAM and LAR

Apart from the flooding algorithms of DREAM and LAR, multi-path algorithms can be based directly on greedy forwarding or planar graph routing. Greedy forwarding can be generalized by selecting the c best neighbors according to the forwarding criterion in each step, instead of considering only one candidate [61]. A multi-path strategy based on face traversals is the concurrent face routing (CFR) [13] algorithm. CFR follows the idea of face routing and duplicates a message whenever a face is encountered that is closer to the destination. Then the duplicate traverses the new face, while the original continues the traversal of the old face.

While multi-path strategies try to increase the chance of reaching a single target, geocasting algorithms deliver a message to multiple targets within a specific geographic area. Flooding-based strategies such as LAR allow to perform geocasting. However, sending a message by unicast into the target area and subsequent flooding with geographic restrictions is not sufficient to guarantee delivery to all target nodes. If the target area is intersected by a void region, i.e. there are two nodes within the target area that are only connected by a path that leaves the target area, then the restricted flooding reaches the first node, but stops at the border of the target area and does not reach the second node. In this case, additional messages have to be sent around the target area to reach the second node. Here, face routing can serve as strategy to traverse the void region and find further nodes within the target area. [79].

A comprehensive overview on geocasting techniqes can be found in the surveys by Jiang and Camp [40], Maihöfer [65] and Stojmenovic [80].

7 Location Service

The third assumption for position-based routing is that the position of the destination node is known. Providing each requesting node with this information is the task of a *location services*.

There are similarities between the problem of locating the destination node and the route discovery problem in ad hoc routing protocols. *Proactive* schemes disseminate routing information before it is needed, whereas *reactive* schemes start the route discovery on demand. Similarly, in the case of a location service the question can be asked, whether location information should be spread proactively or whether a requesting node should look up this information on demand.

Location services can be roughly divided into the following categories:

- Flooding-based location dissemination
- Quorum-based and home-zone-based strategies
- Movement-based location dissemination

Flooding-based location dissemination is fastest way to spread information in the network. It is used in DREAM [4] and LAR [47]. In DREAM each node maintains a location database and distributes its own position proactively throughout the network at regular intervals. In contrast, LAR works reactively: If the destination's position is unknown, the network is flooded with a route request, while the flooded area is restricted to a geographic region where the destination is expected. Flooding-based approaches distribute location information effectively, but at the expense of a high communication overhead.

Quorum-based and *home-zone-based* strategies try to reduce the high communication overhead of flooding. In quorum-based approaches, the location information is held by a group of nodes, which have to be contacted to obtain the target location, whereas in home-zone-based approaches the location is stored by a single node, which is determined by a geographic location. Early approaches for this kind of strategies were proposed by Stojmenovic in [76, 77]. A good example for a quorum-based location service, which uses a geographic criterion for selecting the quorum, is described in [62]. In this scheme the position information is stored along "rows and columns", i.e. a row of nodes having almost the same latitude and a column of nodes with similar longitude store the location information for a specific node. In order to distribute the information, messages are sent along longitudes and latitudes in the network, using geographic routing. A location query can be sent simply in longitudinal or in latitudinal direction in order to find a node of the quorum. For a location service, the nodes of a quorum do not have to be geographically co-located. In the Grid Location Service (GLS) by Li et al. [58] the location servers are distributed according to a hierarchical subdivision of the plane, which is used to assign location information to specific nodes.

In home-zone-based approaches each node has a home-zone with another node that can answer location queries. This is similar to the concept of the home agent in Mobile IP. In order to determine a home zone in a distributed way, a hash function can be used that maps addresses to geographic locations. This is the idea of Geographic Hash Tables (GHT) by Ratnasamy et al. [69]. Given a node and its location as a key-value pair, a hash function maps the key to a geographic location. The node closest to this location then stores the key-value pair. Location queries are routed to the hash location using geographic routing. As the hash location usually does not match exactly a node's location, a face traversal along the nodes surrounding the hash location finally returns the closest node that answers the query.

A *movement-based* dissemination strategy is presented in [30]. Here, the nodes do not spread information over multiple hops. Location information is exchanged only locally, but by the nodes' movement the information is disseminated in the network. Node exchange location information whenever they encounter other nodes. Thus, the information dissemination is provided by mobility. Each node holds a table of locations and timestamps of the last encounter with other nodes. Required that the mobility pattern allows an encounter of all nodes, this information is sufficient to route messages by following the newest information about the target node.

An overview of location services can be found in [78] and [23].

8 Applications

Due to its stateless nature, geographic routing is considered to be superior to topological routing in dynamic and mobile networks. However, without topological information, the routing success depends on the likelihood of encountering local minima and the robustness of the recovery strategy. Many known recovery strategies are limited under practical considerations because they rely on unrealistic assumptions or work inefficiently because of a high communication overhead by traversals or local flooding. Greedy forwarding, especially the contention-based variants, can provide efficient and reactive routing, also in mobile networks. Consequently, these strategies are attractive for vehicular networks and have been used in projects on car-to-car communication. Vehicular networks aim at providing a fast and secure exchange of safety information such as obstacle warnings or lane change warnings or the communication with road side units for the purpose of traffic information and infotainment applications. As cars can easily be equipped with positioning capabilities and wireless transceivers, they provide a suitable platform for geographic routing protocols.

A real-world implementation of a geographic routing protocol was used for inter-vehicle communication in the FleetNet project [31]: Cars were equipped with GPS, a WLAN transceiver and a Linux router. GPSR was used as a routing protocol and target locations were obtained reactively by flooding a request. Experimental results with a small number of cars in a static and mobile setting are described in [32] and [66]. At the same time, the idea of contention-based forwarding [25] was developed within the same project and adopted for vehicular networks [24]. As contention-based forwarding is a pure greedy strategy, which is prone to the local minimum problem, a specific recovery strategy for street scenarios was developed [64]: The Greedy Perimeter Coordinator Routing (GPCR) is based on the idea that the streets, where network nodes reside, form a natural planar graph. Nodes on junctions can serve as coordinators and decide to which junction a packet will be forwarded, while between the junctions a plain greedy scheme can be used. Based on the assumption that street and junctions form a planar graph, a recovery strategy using the right-hand rule can be applied. Whether nodes are on junctions and obtain the role of a coordinator can be determined by using beaconing with 2-hop information or from a evaluating the linear relationship between the neighbors positions statistically.

The work on communication in vehicular networks is continued in the Network-on-Wheels (NOW) project [18], which targets at developing reliable and secure protocols for car-to-car and car-to-infrastructure communication. Within this project, various protocols were developed such as a greedy forwarding with transmit power control [17] to reduce interference, or a protocol to reduce the beaconing load by power control [85].

9 Conclusion

Geographic routing is an elegant alternative to topology-based routing. Since the early approaches of greedy routing in the 1980s and the emergence of face routing in 1999, various techniques have been developed to improve forwarding efficiency and success rate. A combination of greedy forwarding combined with a recovery strategy is still considered the state-of-the-art technique in geographic routing. The most prominent protocols belong to this type and guarantee delivery on certain network structures. The table below gives an overview of the delivery guarantees of some selected protocols.

Protocol	Guaranteed delivery on ...			Localized	Other properties
	GG, RNG	any planar graph	any connected graph		
GFG [10]	•	•	–	•	
GOAFR+ [52] **	•	•	–	•	asympt. optimal
GPSR [43] *	•	–	–	•	
GPVFR [57]	•	–	–	–	
GVG [67]	•	•	semi-closures	–	
CLDP [44]	•	•	•	–	
Face Tracing [88]	•	•	•	–	
GDSTR [56]	•	•	•	–	

*) perimeter routing on its own cannot guarantee delivery, cf. [22]
**) this holds for the closest-point variant, cf. [22]

Table 1: Geographic routing protocols with guaranteed delivery

Many of the described geographic routing protocols rely on geometric properties and assumptions regarding the communication graph. These assumptions enable a theoretical analysis of the efficiency, but they are often violated under practical considerations, e.g. if localization is not precise or if transmission ranges are irregular. This does not imply that geographic routing does not work at all under these conditions, but the basic greedy/face routing protocols loose their delivery guarantees. However, there are techniques to overcome the unit disk graph assumption and to solve problems due to mobility and localization errors.

Greedy forwarding is still an efficient and robust method for geographic routing in dense networks. The development of new greedy strategies has led to cost-efficient methods under realistic physical layer assumptions and for reactive message-efficient routing. Greedy strategies have been evolved towards practical applicability. However, they are prone to the local minimum problem and need to be assisted by a recovery strategy. The development of robust and localized recovery strategies is still subject of ongoing research.

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