# Energy, Congestion and Dilation in Radio Networks

[Extended Abstract]

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## ABSTRACT

We investigate the problem of path selection in radio networks for a given set of sites in two-dimensional space. For some given static point-to-point communication demand we define measures for congestion, energy consumption and dilation that take interferences between communication links into account.

We show that energy optimal path selection for radio networks can be computed in polynomial time. Then, we introduce the diversity $g(V)$ of a set $V \subseteq \mathbb{R}^2$. It can be used to upperbound the number of interfering edges. For real-world applications it can be regarded as $\Theta(\log n)$. A main result of the paper is that a weak $c$-spanner construction as a communication network allows to approximate the congestion-optimal communication network by a factor of $O(g(V)^2)$.

Furthermore, we show that there are vertex sets where only one of the performance parameters congestion, energy, and dilation can be optimized at a time. We show trade-offs lower bounding congestion $\times$ dilation and dilation $\times$ energy. For congestion and energy the situation is even worse. It is only possible to find a reasonable approximation for either congestion or energy minimization, while the other parameter is at least a polynomial factor worse than in the optimal network.

## Categories and Subject Descriptors

C.2.1 [**Computer-communication Networks**]: Network Architecture and Design; F.2.3 [**Analysis of Algorithms and Problem Complexity**]: Tradeoffs between Complexity Measures; G.2.2 [**Discrete Mathematics**]: Graph Theory

## General Terms

Algorithms, Design, Theory

## Keywords

Radio networks, routing, wireless communication

## 1. INTRODUCTION

In this paper we contribute to modeling radio networks, to modeling congestion, energy consumption and dilation for routing in such networks, and to designing routing paths in order to minimize these cost measures. One major insight is the fact that trade-offs are unavoidable: minimizing one measure is only possible at the cost of enlarging another one.

Wireless ad hoc networks consist of nodes that can communicate via short-range wireless connections. Each node can be source, destination and router for data packets, thus no explicit infrastructure is required to set up and maintain an ad hoc radio network. The area of application for radio networks is broad, especially in niches such as search and rescue missions or environmental monitoring. But ad hoc networks can also be used as a last-mile technology to provide access to the Internet in high-populated environments.

In wireless ad hoc networks, energy-expensive long-range connections should be avoided, and the overall distance between two communicating nodes (respectively hop count) should be minimized to achieve low latencies. To use the available network capacity efficiently and to achieve high bandwidths, congested connections should also be avoided by balancing the traffic over all reasonable connections.

These requirements can be expressed using three measurable quantities: congestion, energy and hop count. Traditional routing protocols such as AODV, DSDV and DSR [19] usually chooses the path with the lowest hop count. There also exist power-aware routing protocols that use different metrics (e.g., energy consumed per packet, variance in node power level) to choose the best route in order to extend the lifetime of individual nodes or the whole network [22, 23, 3]. The congestion of a route is usually not regarded directly, but some routing protocols choose routes with the shortest route discovery, assuming that the route with the quickest response is less congested (e.g., SSA [5]). However, to our knowledge, no practical work or theoretical studies exist that consider the interdependencies between these three

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quantities.

In radio networks it is not clear how to choose nodes as communication partners because links can interfere with each other. Our main goal in this paper is to determine the optimal choice of this network given a set of vertices $V \subseteq \mathbb{R}^2$ (Random choices of vertex sets have been investigated in [1, 10]). Hence, we disregard the mobile and dynamic components of ad hoc-networking and determine the optimal static wireless network. We present a general model for congestion, energy and dilation for a given solution of the routing problem of radio networks using one radio frequency. (cf. packet radio network model or more realistic wireless network models, for instance as in [1, 24, 10, 2, 14, 11]). Besides the load the congestion also measures the interferences between edges.

In Section 2 we start our considerations with the paths of all packets solving a routing problem in a radio network. The union of all these paths, called path system, gives a natural definition of the communication network. These paths induce load on the communication links that can interfere with each other. Combining the load and the interferences we achieve an intuitive model for the congestion of an edge of the communication network. Our definition is very similar to those in [1], yet they use a slightly different approach. Likewise in [1] we relate the congestion and the dilation, also known as hop-distance, to the routing time of the routing problem. Then, we define measures for energy consumption, which is important for autonomous nodes that have to "carry their energy".

The main contributions concern path selection in radio networks: Given a set of routing requests, find a routing path so that the congestion, dilation, and/or energy consumption is minimized. We introduce the notion of diversity to describe locations of vertex sets where high interferences are unavoidable. It turns out that if the diversity is small, i.e. all point to point distances differ only by a polynomial factor, then the interferences of communication networks can be kept small. This is key factor for the congestion avoidance analysis in this paper.

In section 3 we present strategies for path selection that provably optimize energy consumption and give a $O(g(V)^2)$-factor approximation of congestion. In section 4, as a main insight, we can conclude that not any two of these measures can be minimized simultaneously. Trade-offs between two measures are unavoidable. Finally, section 5 concludes the work.

## 2. MODELING RADIO NETWORKS

We consider a set $V \subseteq \mathbb{R}^2$ of $n$ radio stations, featuring both transmitter and receivers of one frequency, called sites or vertices, in 2-dimensional Euclidean space. Let $d = \max_{u,v \in V} |u, v|$ denote the geometric diameter of $V$.

As in the model of [18] each node $u \in V$ can adjust its transmission radius to some $r \geq 0$ for sending a packet to a neighbor $v \in V$ in range $r$. Then, the communication network $N = (V, E)$ has the edge $\{u, v\}$, where $|u, v| = r$. To acknowledge this packet the receiving site adjusts its transmission radius to the same radius $r$. The transmission needs a unit time step and the area covered by sending and acknowledging a packet along $e = (u, v) \in E$ is $D(e) = D_r(u) \cup D_r(v)$, where $D_r(u)$ denotes a disk with center $u$ and radius $r$. The edge $e'$ interferes with $e$ if $D(e')$ contains $u$ or $v$ (cp. Figure 1). Since nodes can adjust its transmission power for sending packets, interferences may not be symmetric.

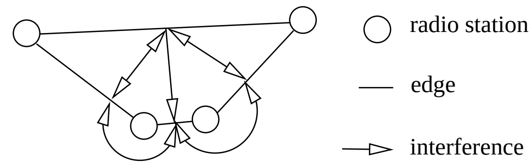


**Figure 1: Stations, edges and induced interferences**

In [9] we extend the definition of the interference number to directed communication. There we allow two communication modes. In the packet routing mode acknowledgment signals are very short and we can neglect its impact on the interferences. When control messages have to be exchanged sending and answering signals are both short, then we have to consider all combination of interferences. The results of this work, especially the trade-offs and the incompatibility of congestion and dilation, apply to both interference models.

We define the set of interfering edges by $\text{Int}(e) := \{e' \in E(N) \mid e' \text{ interferes with } e\}$. Note that sending a packet along $e$ is successful only if no edge from $\text{Int}(e)$ sends concurrently. These interferences of network $N$ describe the directed *interference graph* $G_{Int}(N)$ . Its vertex set are all edges of $N$ and its edges describe all interferences, i.e. $(c, e) \in E(G_{\text{Int}}(N))$ iff $c \in \text{Int}(e)$. The in-degree of an edge in the interference graph is called the *interference number* of a communication link. The maximum interference number of a site $u$ is the maximum interference number of all edges with receiving site $u$. The interference number of the network is the maximum interference number of all edges.

Now consider a routing problem $w : V \times V \to \mathbb{N}$, where $w(u, v)$ packets have to be sent from $u$ to $v$. We subdivide the design of a routing strategy for $w$ into the following steps:

- *Path selection*: Select a system $\mathcal{P}$ of paths $R_p$ from source to destination for the packets $p$ in the graph on $V$. The union of all edges $E_\mathcal{P}$ of the path system gives the links of the communication network $N = (V, E_\mathcal{P})$.

- *Collision avoidance*: As noted above sending a packet along edge $e$ is only successful if no $e' \in \text{Int}(e)$ sends at the same time.

Consider any routing strategy that routes $w$ in $T$ steps using the path system $\mathcal{P}$. Let $\Gamma(e) \subseteq \{1, \ldots, T\}$ denote the time steps in which $e$ sends successfully, then $|\Gamma(e)|$ is just the load $\ell(e)$ of $e$, i.e. the number of packets whose path goes through $e$, and $\sum_{e' \in \text{Int}(e)} \ell(e')$ is the load of all edges interfering with $e$. We combine these quantities and get $\ell(e) + \sum_{e' \in \text{Int}(e)} \ell(e')$ (which is uniquely defined by the path system $\mathcal{P}$) as congestion of the edge $C_\mathcal{P}(e)$. The **congestion of the path system** $\mathcal{P}$ is defined by

$$C_\mathcal{P}(V) := \max_{e \in E_\mathcal{P}} \{C_\mathcal{P}(e)\} \ .$$

We will denote by the dilation $D_\mathcal{P}(V)$ the length of a longest path in $\mathcal{P}$, also known as the hop-distance. By definition the optimal routing time $T$ using $\mathcal{P}$ fulfills $T \geq D_\mathcal{P}(V)$, but also congestion gives a lower bound on the time $T$:

THEOREM 1. *Consider a radio network $M$ with path system $\mathcal{P}$, maximum interference number $I$, and a routing problem $w$ with dilation $D$ and congestion $C$. Let $T$ be its optimal routing time, when the path system $\mathcal{P}$ is used. The following holds.*

1. $T \;\geq\; \max\{C/12, D\} = \Omega(C + D)$
2. *It exists an offline routing protocol with routing time $O(C + D \cdot I)$, with high probability.*
3. *There is an online routing protocol that needs routing time $O(C + D \cdot I \cdot \log n)$, w.h.p.*

PROOF. 1. Let $e = (u, v)$ be an edge with maximum congestion $C$. We partition the plane into 6 regions $R_1, \ldots, R_6$ with center at $u$ by six half-lines starting at $u$ where the angle between neighbored half-lines is $\pi/3$. Similarly we consider the analogous partitioning $R_7, \ldots, R_{12}$ with $v$ as the starting point of the 6 half-lines.

Define

$$E_i := \{\{p, q\} \mid (p \in R_i \vee q \in R_i) \wedge \{p, q\} \in \mathrm{Int}(e)\} \ .$$

Note that by a straight-forward geometric argument for two edges $e', e'' \in E_i$ it holds either $e' \in \mathrm{Int}(e'')$ or $e'' \in \mathrm{Int}(e')$. Therefore, all transmissions over edges in $E_i \cup \{e\}$ have to be done sequentially. Let $\ell_i := \ell(e) + \sum_{e' \in E_i} \ell(e')$. Then, $\sum_{i=1}^{12} \ell_i \geq C$. Hence,

$$T \;\geq\; \max_{i \in [12]}\{\ell_i\} \;\geq\; \frac{1}{12}\sum_{i=1}^{12} \ell_i \;\geq\; \frac{C}{12} \ .$$

The upper bounds of 2. and 3. can be proved using the same arguments as shown in Theorem 2.12 and Theorem 2.13 of [1]. Note that in [1] the notion dilation differs from our approach. □

The variable choice of the transmitter power allows to reduce the energy consumption, saving on the tight resources of batteries in portable radio stations and reducing interferences. Theoretically, the energy needed to send over a distance of $r$ is given by $O(r^2)$. It turns out that in practice one can model the energy by $O(r^4)$ or even $O(r^5)$. Throughout this paper we model energy costs by $O(r^2)$. However, all results besides theorem 3 in this paper can be easily transferred for higher exponents.

We distinguish two energy models. In the first model, called **unit energy model**, we assume that maintaining a communication link $e$ is proportional to $O(|e|^2)$, where $|e|$ denotes its Euclidean length. Therefore, the unit energy *U-Energy* used by radio network $N$ is given by

$$\text{U-Energy}_{\mathcal{P}}(V) := \sum_{e \in E_{\mathcal{P}}(N)} |e|^2 \ .$$

The **flow energy model** reflects the energy actually consumed by transmitting all packets. Here, the power consumption of a communication link is weighted by the actual load $\ell(e)$ on an edge $e$:

$$\text{F-Energy}_{\mathcal{P}}(V) := \sum_{e \in E_{\mathcal{P}}(N)} \ell(e)|e|^2 \ .$$

In this paper we focus on the question: Given some sites which path selection is best possible to obtain small congestion, low energy consumption and small dilation. Clearly, the optimal network for hop-distance is the complete graph. Hence, we investigate only energy and congestion.

# 3. MINIMIZING ENERGY AND CONGESTION

## 3.1 Energy

The unit energy of a path system for a radio network is defined as the energy consumption necessary to deliver one packet on each communication link. It turns out that the minimal spanning tree optimizes unit energy. Note that the hardness results shown in [14, 4] do not apply because in our model the transmission radii are adjusted for each packet.

THEOREM 2. *The minimal spanning tree is an optimal path system for a radio network with respect to the unit energy.*

PROOF. Consider the graph defined by all edges $E \subseteq V \times V$ with edge weight $|e|^2$. The minimum energy network can be constructed using Prim's or Kruskal's algorithm for minimum spanning tree. Note that the decisions in this algorithm are based on comparison of the length of some edges $e$ and $e'$, i.e. $|e| \leq |e'|$. Thus, the minimal network for energy is also the minimum spanning tree for Euclidean distances. □

For the flow energy model, the minimal network is not necessarily a tree. However, one can compute the minimal flow energy network in polynomial time. In consideration of the flow energy we use the gabriel graph [6] that consists of all edges $(u, v)$ such that the open disk using $|u, v|$ as diameter does not contain any node from $V$. Then the following holds:

THEOREM 3. *For a given vertex set $V$ a sub-graph of the Gabriel Graph is an optimal path system for a radio network with respect to the flow energy.*

PROOF. If in the interior of the circle defined by the diameter $(u, v)$ there exists a vertex $w$, then the edges $(u, w), (w, v)$ need less energy than the original edge. This follows by the Theorem of Thales. Therefore, one can add an edge into the communication network iff in there are no sites in the interior of its circle, see Fig. 2. This matches the definition of a Gabriel graph of $V$.

For two vertices $u$ and $v$ the sub-graph providing the lowest energy for routing information from $u$ to $v$ is given by the shortest path in the Gabriel graph if the length of an edge is redefined by $|e|^2$. The flow energy of the optimal network consists of a linear combination of these lowest-energy-paths between pairs. Using an all-pair-shortest-path algorithm gives the optimal network. □

Note, that there are situations where edges of the Gabriel-graph can be replaced by less energy-consuming paths, even if no site lies inside the disk described by the edge. Then, the edge of the Gabriel graph is not part of any energy optimal route.

## 3.2 Congestion

### *3.2.1 Diversity of a Vertex Set*

Sometimes the location of the radio stations does not allow any routing without incurring high congestion. Consider a vertex set $V = \{v_1, \ldots, v_n\}$ on a line, with distances

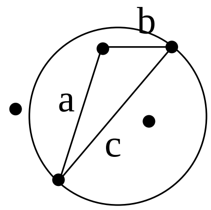


**Figure 2: Communication on an edge $c$ is more expensive with regard to unit energy than communication on the edges $a$ and $b$ ($a^2 + b^2 < c^2$)**

$|v_i, v_{i+1}| = 2^i$. The edge $(v_i, v_{i+1})$ interferes with all edges $(v_j, v_{j+1})$ for $j \leq i$, see Fig. 3. Therefore the interference number of the network is $n-1$. Suppose only $v_1$ and $v_n$ want to communicate, then the better solution for congestion is to disconnect all interior points and to realize only the edge $(v_1, v_n)$. Of course this is not an option when interior nodes need to communicate.

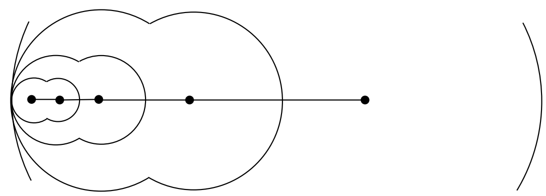

**Figure 3: The high diversity of the vertex set causes many interferences, resulting in high congestion**

It turns out that a determining parameter for the realization of optimal communication networks for radio networks, is the number of magnitudes of distances. Distances have different magnitude if they differ more than a factor of 2.

Definition 1. *The diversity $g(V)$ of a point set $V$ in Euclidean space is defined by*

$$g(V) := |\{m \mid \exists u, v \in V \ : \ \lfloor \log |u, v| \rfloor = m\}| \ .$$

Note that in the above scenario we observe the almost maximum diversity of $n$ (and a high interference number). For point sets $V$ on the line with small diversity the interference number is small, too. It is easy to see, that the interference number for a vertex set $V$ on the line is at most $O(g(V))$.

Lemma 1. *The diversity of $n$ points in $\mathbb{R}^2$ is at least $\Omega(\log n)$ and at most $O(n)$. For a point set randomly distributed in a square of $\mathbb{R}^2$ the diversity is $O(\log n)$ with high probability (i.e., $1 - n^{-c}$ for any fixed constant $c > 0$). Furthermore,*

$$g(V) \leq 1 + \log \frac{\max_{u,v \in V} ||u, v||_2}{\min_{u,v \in V \wedge u \neq v} ||u, v||_2} \ .$$

The last inequality follows directly from the definition. It implies logarithmic diversity for random point sets since the probability to choose a vertex within a $n^{\frac{-c-1}{2}}$-neighborhood of another can be bound by $n^{-c-1}$. Hence, for all vertices the probability that $g(V) \geq (\frac{c+1}{2}) \log n$ can be bounded by at most $n^{-c}$. The proofs for the upper and lower bounds can be found in [20].

There are many reasons why in the real world the diversity can always be estimated by $O(\log n)$, e.g. the accuracy of determining locations; and the ratio between the physical size of a radio station and its transmitting range.

### 3.2.2 Approximating Congestion

To approximate congestion-optimal communication networks for radio networks we will use the *Hierarchical Layer Graph* with bounded degree introduced in [9]. Adopting ideas from clustering [7, 8] and generalizing an approach of [1] we present a graph consisting of $w$ layers $L_0, L_1, \ldots, L_w$. The union of all this graphs gives the Hierarchical Layer graph. The lowest layer $L_0$ contains all vertices $V$. The vertex set of a higher layer is a subset of the vertex set of a lower layer until in the highest layer there is only one vertex, i.e. $V = V(L_0) \supseteq V(L_1) \supseteq \cdots \supseteq V(L_w) = \{v_0\}$.

The crucial property of these layers is that in each layer $L_i$ vertices obey a minimum distance: $\forall u, v \in V(L_i) \ : \ |u, v| \geq r_i$. Furthermore, all nodes in the next-lower layer must be covered by this distance: $\forall u \in V(L_i) \quad \exists v \in V(L_{i+1}) \quad : |u, v| \leq r_{i+1}$. Our construction uses parameters $\alpha \geq \beta > 1$, where for some $r_0 < \min_{u,v \in V} |u, v|$ we use radii $r_i := \beta^i \cdot r_0$ and we define in layer $L_i$ the edge set $E(L_i)$ by $E(L_i) := \{(u, v) \mid u, v \in V(L_i) \ \wedge \ |u, v| \ \leq \ \alpha \cdot r_i\}$.

Clearly, for a vertex set $V$ with diversity $g(V)$ we have a maximum number of $w = O(g(V))$ layers. We will see that the *weak c-spanner* property has implication for minimizing congestion. Note, that by the following definition a $c$-spanner $G = (V, E)$ is also a weak $c$-spanner.

Definition 2. *A graph $G = (V, E)$ is a $c$-spanner, if for all $u, v \in V$ there exists a (directed) path $p$ from $u$ to $v$ with $|p| \leq c \cdot |u, v|$. $G$ is a weak $c$-spanner, if for all $u, v \in V$ there exists a path $p$ from $u$ to $v$ which is covered by a disk of radius $c \cdot |u, v|$ centered at $u$.*

Theorem 4. *[9] If $\alpha > 2\frac{\beta}{\beta-1}$ the Hierarchical Layer Graph is a $c$-spanner (and therefore a weak $c$-spanner) for $c = \max\left\{\beta\frac{\alpha(\beta-1)+2\beta}{\alpha(\beta-1)-2\beta}, \frac{\alpha}{\beta}\right\}$.*

Lemma 2. *For a vertex set $V$ with diversity $g(V)$ the interference number of the Hierarchical Layer Graph is bounded by $O(g(V))$.*

Proof. The interference number of the Hierarchical Layer Graph is bounded by the number of layers. By definition we have a constant number of interferences in each layer and therefore the number of layers is bounded by $w = O(g(V))$. $\square$

A typical feature of radio communication is that transmitting information blocks a region for other transmission. We formalize this observation and define the capacity of a region following a similar approach presented in [10]. Let $A(R)$ denote the area of a geometric region $R$.

Definition 3. *The capacity $\kappa(R)$ of a geometric region $R$ is defined as follows: If in every point of $R$ the same set of edges $E$ interfere then $\kappa(R) := \sum_{e \in E} \ell(e) \cdot A(R)$ , where $A(R)$ denotes the area of $R$. Such a region is called elementary. Otherwise partition $R$ into elementary regions $R_1, \ldots, R_m$ and define $\kappa(R) := \sum_{i=1}^{m} \kappa(R_i)$ .*

This definition implies the following relationship between capacity, area and congestion.

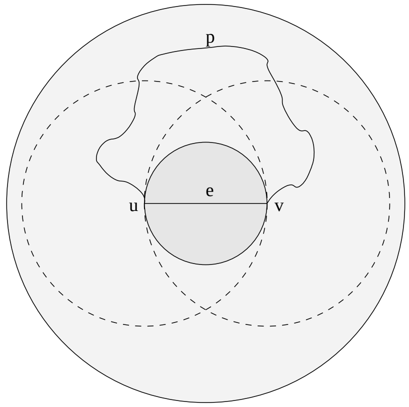


**Figure 4: The edge $e$ interferes with other edges (at least) within the central disk. Its information is rerouted on $p$, lying completely within the outer-disk with radius $(c - \frac{1}{2})|e|$**

LEMMA 3. *Let $R$ be a region and $C$ the congestion of a path system $\mathcal{P}$. Then, the capacity of $R$ is bounded by $\kappa(R) \leq A(R) \cdot C$.*

Every edge $e$ with load $\ell(e)$ has a certain impact on the capacity of the area covered by the radio-network.

LEMMA 4. *An edge $e$ with load $\ell(e)$ occupies the capacity $c\ell(e)|e|^2$ for a constant $c > 0$.*

The proof follows from the definition of the interference area.

LEMMA 5. *Let $C^*$ be the congestion of the congestion-optimal path system $\mathcal{P}^*$ for a vertex set $V$. Then, every weak $c$-spanner $N$ can host a path system $\mathcal{P}'$ such that the induced load $\ell(e)$ in $N$ is bounded by $\ell(e) \leq c' g(V)\ C^*$ for a positive constant $c'$.*

PROOF. Given a path $p$ of the path system $\mathcal{P}^*$, we replace every edge $e = (u, v)$ that does not exist in the weak $c$-spanner $N$ with a path $p$ from $u$ to $v$ in $N$ such that the new route lies completely inside a disk $D_c(e)$ of radius $(c - \frac{1}{2})|u, v|$ and center $\frac{1}{2}(u + v)$.

For the path system $\mathcal{P}^*$ there may have been interferences between $e$ and other edges. For simplicity we underestimate the area where $e$ can interfere other communication by the disk $D_1(e)$ with center $\frac{1}{2}(u + v)$ and radius $\frac{1}{2}|u, v|$ (see Fig. 4).

We want to describe the impact of rerouting of all edges in $E(N^*)$ to a specific edge $e_0 \in E(N)$ in the $c$-spanner $N$. If this edge $e_0 = (u_0, v_0) \in E(N)$ transmits the traffic of a detour of an edge $e = (u, v) \in E(N^*)$, then the distance between the central points $z_0 := \frac{1}{2}(u_0 + v_0)$ of $e_0$ and $z := \frac{1}{2}(u + v)$ is bounded by $|z_0, z| \leq (c - \frac{1}{2})|e|$.

Now consider the edge set $E_{i,e_0} \subseteq E(N^*)$ of edges $e$ with length $|e| \in [2^i, 2^{i+1}]$ for $i \in \mathbb{Z}$ which reroute their traffic to $e_0$. Their center points are located inside a disk with radius $(c - \frac{1}{2})2^{i+1} \leq 2^{i+1}c$ and center $z_0$. The interference area of every edge $e$ is described by $D_c(e)$. It occupies an area of at least $\pi 2^{2i}$, which lies completely inside a disk $D$ with radius $2^{i+1}(c + 1)$ and center $z_0$. The area of $D$ is $\pi 2^{2i+2}(c + 1)^2$.

Lemma 4 shows that every edge $e$ reduces the capacity in $D$ by at least $c''\ell(e)2^{2i}$. Because of Lemma 3, the over-all capacity of $C$ is at most $\kappa(D) = \pi 2^{2i+2}(c+1)^2 C^*$. Therefore we have for the sum of the loads $\ell(e)$ for $e \in E_{i,e_0}$ that $\sum_{e \in E_{i,e_0}} \ell(e) \leq \pi 4(t + 1)^2 C^*/c''$ . By definition there are at most $g(V)$ non-empty sets $E_{i,e_0}$. This implies for the sum of loads $\ell(e)$ of the set $E_{e_0} \subseteq E(N^*)$:

$$\sum_{e \in E_{e_0}} \ell(e) \leq g(V)4(t + 1)^2 \pi C^*/c'' = c' C^* g(V),$$

where $c' := 4(t + 1)^2 \pi / c''$. $\square$

THEOREM 5. *Let $\mathcal{P}^*$ be the congestion optimal path system for the vertex set $V$. Then the Hierarchical Layer Graph contains a path system $\mathcal{P}$ with congestion $O(g(V)^2 C_{\mathcal{P}^*}(V))$.*

PROOF. From Theorem 4 it follows that the Hierarchical Layer Graph is a weak $c$-spanner. Therefore we can use Lemma 5 to show that there exists a routing such that the load of an edge $e$ is bounded by $\ell(e) \leq c' g(V) C_{\mathcal{P}^*}(V)$. Lemma 2 shows that the interference number of the network is bounded by $O(g(V))$. So, this implies that $C_{\mathcal{P}}(V) = O(g(V)^2 C_{\mathcal{P}^*}(V))$. $\square$

Since in practice the diversity can be seen as a logarithmic term, the Hierarchical Layer Graph provides a $O((\log n)^2)$-approximation for congestion.

# 4. TRADE-OFFS

We have seen efficient ways for selecting paths to optimize energy and approximate congestion. One might wonder whether an algorithm can compute a path system for a radio network optimizing energy, congestion and dilation at the same time. It turns out that this is not the case.

## 4.1 Congestion versus Dilation

For a vertex set $G_n$ placed on the crossings of a $\sqrt{n} \times \sqrt{n}$-grid the best choice to minimize congestion is to connect grid points only to their neighbors given the demand $w(u, v) = W/n^2$ for all vertices (Fig. 5). Then the congestion is $O(W/\sqrt{n})$ and the dilation is given by $O(\sqrt{n})$. In [10] it is shown that such a congestion is best possible in a radio network. A fast realization is given by a tree featuring a hop-distance of $O(\log n)$ and congestion $O(W \log n)$ (Such a tree-construction for the Cost-distance problem is presented in [21]). In both cases we observe $C_{\mathcal{P}}(G_n) D_{\mathcal{P}}(G_n) \geq \Omega(W)$. This also is true for any other path selection:

THEOREM 6. *Given the grid vertex set $G_n$ with traffic $W$ then for every path system $\mathcal{P}$ the following trade-off between dilation $D_{\mathcal{P}}(G_n)$ and congestion $C_{\mathcal{P}}(G_n)$ exists:*

$$C_{\mathcal{P}}(G_n) \cdot D_{\mathcal{P}}(G_n) \geq \Omega(W) \ .$$

PROOF. For $n = 9p^2$ partition the grid into three $p \times 3p$ rectangle shaped vertex sets $V_1, V_2, V_3$, such that $V_1$ contains all left vertices, $V_3$ all right vertices and $V_2$ the vertices in the middle.

We consider only an $\frac{1}{9}$th of the demand starting at $V_1$ heading for vertices in $V_3$. Let $D \leq 3p$ be the dilation of the network and $p_{i,j}$ denote the route from vertex $v_i$ to vertex

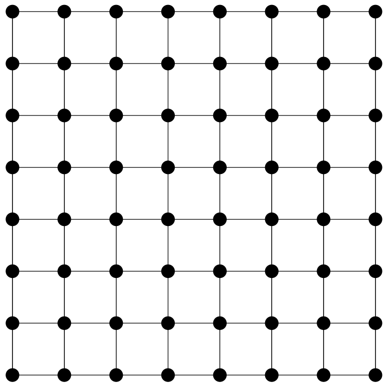

**Figure 5: The grid $G_n$**

$v_j$. Let $r(p_{i,j}) = w(u_i, u_j)$ denote the information flow on path $p_{i,j}$.

Consider two vertices $v_i \in V_1$ and $v_j \in V_3$. Then the path $p_{i,j}$ has at most $D_{\mathcal{P}}(G_n)$ edges. The induced capacity $\kappa(p_{i,j})$ of the path $p_{i,j}$ is at least $\kappa(p_{i,j}) \geq c_1 \ell(p_{i,j}) \sum_{e \in p_{i,j}} |e|^2$ . This term is minimized if the path uses the maximum possible number $D_{\mathcal{P}}(G_n)$ of edges with equal length of at least $\frac{d}{3D_{\mathcal{P}}(G_n)}$. Then, we have $\kappa(p_{i,j}) \geq \frac{c_1 d^2 W}{9n^2 D_{\mathcal{P}}(G_n)}$ .

The sum of the capacity over all paths cannot extend the capacity of the $(3d) \times (3d)$-square containing all possible interference areas. This gives:

$$\sum_{v_i \in V_1} \sum_{v_j \in V_3} \kappa(p_{i,j}) \leq 9d^2 \ .$$

Combining the inequalities states the claim, since we get

$$C_{\mathcal{P}}(G_n) D_{\mathcal{P}}(G_n) \geq \frac{c_1}{81} W \ .$$

□

## 4.2 Dilation versus Energy

The simplest location of sites is the line vertex set $L_n$ as investigated in [14], see Fig. 6. Here all vertices $L_n = \{v_1, \ldots, v_n\}$ are placed on a line with equal distances $|v_i, v_{i+1}| = \frac{d}{n}$. Only the leftmost and the rightmost node want to exchange messages, i.e. $w(v_1, v_n) = W$ and $w(v, w) = 0$ for all other pairs $(v, w)$. The energy-optimal network for unit and flow energy is the path $(v_1, v_2, \ldots, v_n)$, given the unit energy U-Energy$_{\mathcal{P}}(L_n) = \frac{d^2}{n}$, the flow energy F-Energy$_{\mathcal{P}}(L_n) = \frac{d^2 W}{n}$ and the dilation $n$.

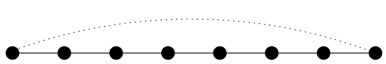

**Figure 6: The line $L_n$**

The fasted network realizes only the edge $(v_1, v_n)$ with hop-distance 1 and unit energy $d^2$ (and flow energy $Wd^2$). There are path systems that can give a compromise between these extremes. However, it turns out that the product of dilation and energy cannot be decreased:

THEOREM 7. *Given the vertex set $L_n$ with diameter $d$ then for every path system $\mathcal{P}$ the following trade-offs between dilation $D$ and unit energy* U-Energy *(resp. flow energy* F-Energy*) exist:*

$$\begin{aligned} D_{\mathcal{P}}(L_n) \cdot \textit{U-Energy}_{\mathcal{P}}(L_n) &\geq \Omega(d^2) \ , \\ D_{\mathcal{P}}(L_n) \cdot \textit{F-Energy}_{\mathcal{P}}(L_n) &\geq \Omega(d^2 W) \ . \end{aligned}$$

PROOF. Let $m$ be the number of edges of the longest path of the radio network (wlog we assume that there are only edges with non-zero information flow $\ell(e) > 0$). For the unit energy model we can assume that there is only a path $p$ from $v_1$ to $v_n$ (because introducing more edges needs additional energy without decreasing the dilation). We have to minimize U-Energy$_{\mathcal{P}}(p) := \sum_{i=1}^{m} (\ell_i)^2$ defined by the edge lengths $\ell_1, \ldots, \ell_m$, where $\sum_{i=1}^{m} \ell_i = d$. Clearly, the energy sum is minimal for $\ell_1 = \ell_2 = \cdots = \ell_m = \frac{d}{m}$ giving U-Energy$_{\mathcal{P}}(p) \cdot D_{\mathcal{P}}(p) \geq d^2$ .

The bound for the flow energy follows analogously. □

## 4.3 The Incompatibility of Congestion and Energy

We will show that for some vertex sets congestion and energy are incompatible. This is the worst oocurence of a trade-off-situation since there is no possible compromise between energy and congestion.

The vertex set $U_{\alpha,n}$ for $\alpha \in [0, \frac{1}{2}]$ consists of two horizontal parallel line graphs $L_{n^\alpha}$. Neighbored (and opposing) vertices have distance $\frac{d}{n^\alpha}$. There is only demand $W/n^\alpha$ between the vertical pairs of opposing vertices of the line graphs. The rest of the $n - n^{-\alpha}$ vertices are equidistantly placed between the vertices of each line graph and the leftmost vertical pair of vertices (see Fig. 7).

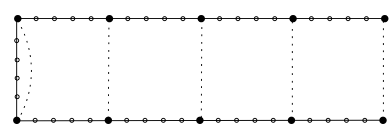

**Figure 7: Vertex set $U_\alpha$**

The minimum spanning tree consists of $n$ vertices where all edges have length $\Theta(dn^{-1})$. This results in a total unit energy of

$$\text{U-Energy}_{\text{MST}}(U_{\alpha,n}) = O(d^2 n^{-1})$$

and congestion

$$C_{\text{MST}}(U_{\alpha,n}) = O(W) \ .$$

The flow energy of the (same) minimum network is given by

$$\text{F-Energy}_{\text{MST}}(U_{\alpha,n}) = O(W d^2 n^{-1}) \ .$$

The congestion optimal path system $\mathcal{P}'$ connects only vertices with non-zero demand. Its congestion is

$$C_{\mathcal{P}'}(U_{\alpha,n}) = O(W n^{-\alpha})$$

and its unit energy is

$$\text{U-Energy}_{\mathcal{P}'}(U_{\alpha,n}) = O(d^2 n^{-\alpha}) \ .$$

The flow energy is given by

$$\text{F-Energy}_{\mathcal{P}'}(U_{\alpha,n}) = O(W n^{-2\alpha} d^2) \ .$$

LEMMA 6. *For $\alpha \in [0, \frac{1}{2})$ and the vertex set $U_{\alpha,n}$ with diameter $d$ let $x \in \{0, \ldots, n^\alpha\}$ be the number of edges of length $dn^{-\alpha}$ of a path system for the radio network and let $f \in [0, W]$ be the information flow on these edges. Then, we have for the congestion $C$, unit energy and flow energy:*

$$\begin{aligned}
\textit{U-Energy}_{\mathcal{P}}(U_{\alpha,n}) &\geq \frac{d^2}{n} + x\left(\frac{d^2}{n^{2\alpha}} - \frac{d^2}{n^{1+\alpha}}\right), && (1)\\
C_{\mathcal{P}}(U_{\alpha,n}) &\geq \frac{W}{x+1}, && (2)\\
\textit{F-Energy}_{\mathcal{P}}(U_{\alpha,n}) &\geq f\frac{d^2}{n^{2\alpha}} + (W-f)\left(\frac{d^2}{n} - \left\lfloor \frac{f}{n^\alpha} \right\rfloor \frac{d^2}{n}\right) && (3)\\
C_{\mathcal{P}}(U_{\alpha,n}) &\geq \frac{f}{n^\alpha} + (W-f). && (4)
\end{aligned}$$

PROOF. The minimum unit energy network is given by the MST which is a U-shaped path. Note that no shortcut within the left and right horizontal bars of this path can reduce energy or congestion. Therefore the only reasonable choice for an edge is to connect some $(x)$ of the horizontal vertices (and possibly to disconnect a route to a vertical neighbors). Adding the vertical channel implies additional energy consumption of $\frac{d^2}{n^{2\alpha}}$. For $x+1$ horizontal routes (including the original low energy route) the best choice is to fairly distributed the traffic.

For the flow energy the argument is analogous. □

THEOREM 8. *There exists a vertex set $V$ with a path system minimizing congestion to $C^*$, and another path system optimizing unit energy by U-Energy$^*$ and minimal flow energy by F-Energy$^*$ such we have for any path system $\mathcal{P}$ on this vertex set $V$ we have*

$$\begin{aligned}
C_{\mathcal{P}}(V) &\geq \Omega(n^{1/3}C^*) \quad \textit{or}\\
\textit{U-Energy}_{\mathcal{P}}(V) &\geq \Omega(n^{1/3}\,\textit{U-Energy}^*),\\
C_{\mathcal{P}}(V) &\geq \Omega(n^{1/3}C^*) \quad \textit{or}\\
\textit{F-Energy}_{\mathcal{P}}(V) &\geq \Omega(n^{1/3}\,\textit{F-Energy}^*).
\end{aligned}$$

PROOF. The claim follows directly by Lemma 6 using the graph $V = U_{1/3,n}$. □

Hence, there is no hope that routing in wireless networks can optimize more than one parameter at a time. The wireless network designer has to decide infavor of small congestion or low energy consumption.

# 5. CONCLUSIONS AND FURTHER WORK

The main difference between wired networks and radio networks is that the choice of communication links in wireless networks influences the quality of the edges. We model the type of influences by the interference graph, which gives a very general description how links can interfere. If the sending and receiving characteristics of the radio stations are known, this interference graph can be described by the geometric properties like the location of sites and transmitter power.

However, the main difference is still that choosing a certain communication link for some time decreases the ability of transmitting information in some other parts of the radio network. Since the analysis of point-to-point communication (or permutation networks) in wireless networks is relatively young (see [1]), we start our investigation with a static simplified model: The point-to-point communication and the location of the sites is fixed. You can also see this model as a snapshot of a more dynamic model (where research has just begun [2]).

We investigate the question what the optimal choice of communication links is to achieve the best possible network. We measure the quality by congestion, energy and hop-distance. Given a path system for the packets we present a sound definition of congestion, which takes into account the actual information flow, i.e. load, over a link and possible interferences of other links.

A probabilistic solution for solving interferences has been presented if the network parameters are known [1]. We show how this algorithm can be applied to our setting. Further, we relate routing time to our notion of congestion and dilation, which is the maximum number of edges of a route.

We prove that for our notion of energy (depending on the packet flow) the optimal path system can be computed in polynomial time. Furthermore, we prove that a weak $c$-spanner construction for the communication networks allow path systems with small congestion. Concretely, we show an approximation of a factor of $O(g(V)^2)$ of the minimal congestion, where $g(V)$ denotes the diversity of the vertex set. We introduce this measure to characterize malformed vertex locations. For practical applications we have $g(V) = \Theta(\log n)$, e.g. if $|V| = n$ and if the vertex set is random, or if the ratio of maximum and minimum distance of nodes is at most polynomial (These results are summarized in Table 1).

However there are situations where it is not possible to optimize two of these measures at the same time, see Table 2. We prove trade-off results for congestion versus dilation and energy versus dilation. For congestion and energy we show that every path system trying to approximate the congestion within a smaller factor than $o(n^{1/3})$ of the optimal congestion, suffers under an increased energy consumption of at least a factor of $\Omega(n^{1/3})$, and vice versa. Hence, energy and congestion minimization in radio networks are incompatible tasks.

Another possibility to decrease interferences is to use multiple frequencies, (as done in Bluetooth [16] or IEEE 802.11 [12]). As long as number $f$ of frequencies is small (which is the case in practice, because of governments' regulation of all frequency spectra) this may improve the congestion by this factor $f$. However, using frequency hopping cannot completely resolve the shown trade-off and incompatibility problems. Besides the standard model of omni-directional communication we are currently investigating a sector model where sender and receiver can focus signals (e.g. infrared). Such sector communication is a special case of so-called space multiplexing techniques to increase the network capacity (e.g. by using directional antennas [15]). The techniques of the results shown in this paper can be easily transferred to such a model [9].

At the moment we are working on the implementation of a prototype communication system based on infrared directed communication. The prototype will be allowed to submit data in a fixed number of different directions and to adjust the transmission power in each sector separately. It can be used as an extension module for the mobile mini robot Khepera ([17, 13]). Thus, realistic scenarios for ad hoc networks can be reproduced by performing experiments

| | Congestion | Dilation | Unit Energy | Flow Energy |
|---|---|---|---|---|
| Structure | HL Graph | Complete Network | MST | Gabriel Sub-Graph |
| Approx.-factor | $O(\log^2 n)$ | optimal | optimal | optimal |

**Table 1: Approximation results for logarithmic diversity**

| | Dilation | Congestion |
|---|---|---|
| Congestion | $C_{\mathcal{P}}(V) \cdot D_{\mathcal{P}}(V) \geq \Omega(W)$ | — |
| Unit Energy | $D_{\mathcal{P}}(V) \cdot \mathrm{UE}_{\mathcal{P}}(V) \geq \Omega(d^2)$ | $C_{\mathcal{P}}(V) \geq \Omega(n^{1/3} C^*_{\mathcal{P}}(V))$ or $\mathrm{UE}_{\mathcal{P}}(V) \geq \Omega(n^{1/3} \mathrm{UE}^*_{\mathcal{P}}(V))$ |
| Flow Energy | $D_{\mathcal{P}}(V) \cdot \mathrm{FE}_{\mathcal{P}}(V) \geq \Omega(d^2 W)$ | $C_{\mathcal{P}}(V) \geq \Omega(n^{1/3} C^*_{\mathcal{P}}(V))$ or $\mathrm{FE}_{\mathcal{P}}(V) \geq \Omega(n^{1/3} \mathrm{FE}^*_{\mathcal{P}}(V))$ |

**Table 2: Trade-Offs and Incompatibilities on network parameters**

with these mini robots. Beside computer simulations [25], this enables us to validate our communication strategies under practical conditions. Such a network is technically more complicated, but our goal is to show that it is possible to set up a geometric spanner graph as a communication network. Notably, this paper shows that such geometric spanners always provide good solutions for congestion minimization in radio networks.

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