Wireless Sensor Networks

17th Lecture

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CoNe Freiburg

University of Freiburg
Computer Networks and Telematics
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Goals of this chapter

- **Means for a node to determine its physical position (with respect to some coordinate system) or symbolic location**
- **Using the help of**
  - Anchor nodes that know their position
  - Directly adjacent
  - Over multiple hops
- **Using different means to determine distances/angles locally**

Overview

- **Basic approaches**
- *Trilateration*
- **Multihop schemes**

Trilateration

- **Assuming distances to three points with known location are exactly given**
- **Solve system of equations (Pythagoras!)**
  - (x_i, y_i) : coordinates of ***anchor point*** i, r_i distance to anchor i
  - (x_u, y_u) : unknown coordinates of node

$$(x_i - x_u)^2 + (y_i - y_u)^2 = r_i^2 \text{ for } i = 1, \ldots, 3$$

  - Subtracting eq. 3 from 1 & 2:

$$(x_1 - x_u)^2 - (x_3 - x_u)^2 + (y_1 - y_u)^2 - (y_3 - y_u)^2 = r_1^2 - r_3^2$$
$$(x_2 - x_u)^2 - (x_2 - x_u)^2 + (y_2 - y_u)^2 - (y_2 - y_u)^2 = r_2^2 - r_3^2.$$

  - Rearranging terms gives a linear equation in (x_u, y_u)!

$$2(x_3 - x_1)x_u + 2(y_3 - y_1)y_u = (r_1^2 - r_3^2) - (x_1^2 - x_3^2) - (y_1^2 - y_3^2)$$
$$2(x_3 - x_2)x_u + 2(y_3 - y_2)y_u = (r_2^2 - r_2^2) - (x_2^2 - x_3^2) - (y_2^2 - y_3^2)$$

Trilateration as matrix equation

- **Rewriting as a matrix equation:**

$$2 \begin{bmatrix} x_3 - x_1 & y_3 - y_1 \\ x_3 - x_2 & y_3 - y_2 \end{bmatrix} \begin{bmatrix} x_u \\ y_u \end{bmatrix} = \begin{bmatrix} (r_1^2 - r_3^2) - (x_1^2 - x_3^2) - (y_1^2 - y_3^2) \\ (r_2^2 - r_2^2) - (x_2^2 - x_3^2) - (y_2^2 - y_3^2) \end{bmatrix}$$

- **Example:** **$(x_1, y_1) = (2,1)$, $(x_2, y_2) = (5,4)$, $(x_3, y_3) = (8,2)$, $r_1 = 10^{0.5}$, $r_2 = 2$, $r_3 = 3$**

$$2 \begin{bmatrix} 6 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_u \\ y_u \end{bmatrix} = \begin{bmatrix} 64 \\ 22 \end{bmatrix}$$

$\rightarrow (x_u, y_u) = (5,2)$

Trilateration with distance errors

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- What if only distance estimation $r_i' = r_i + \varepsilon_i$ available?
- Use multiple anchors, overdetermined system of equations

$$2\begin{bmatrix} x_n - x_1 & y_n - y_1 \\ \vdots & \vdots \\ x_n - x_{n-1} & y_n - y_{n-1} \end{bmatrix} \begin{bmatrix} x_u \\ y_u \end{bmatrix} = \begin{bmatrix} (r_1^2 - r_n^2) - (x_1^2 - x_n^2) - (y_1^2 - y_n^2) \\ \vdots \\ (r_{n-1}^2 - r_n^2) - (x_{n-1}^2 - x_n^2) - (y_{n-1}^2 - y_n^2) \end{bmatrix}$$

- Use (x_u, y_u) that minimize mean square error, i.e, $||\mathbf{A}\mathbf{x} - \mathbf{b}||_2$

Minimize mean square error

- **Look at square of the of Euclidean norm expression (note that for all vectors v)** $||\mathbf{v}||_2^2 = \mathbf{v}^{\mathrm{T}}\mathbf{v}$

$$||\mathbf{Ax} - \mathbf{b}||_2^2 = (\mathbf{Ax} - \mathbf{b})^{\mathrm{T}}(\mathbf{Ax} - \mathbf{b}) = \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{Ax} - 2\mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{b} + \mathbf{b}^{\mathrm{T}}\mathbf{b}$$

- **Look at derivative (gradient) with respect to x, set it equal to 0:**

$$2\mathbf{A}^{\mathrm{T}}\mathbf{Ax} - 2\mathbf{A}^{\mathrm{T}}\mathbf{b} = 0 \Leftrightarrow \mathbf{A}^{\mathrm{T}}\mathbf{Ax} = \mathbf{A}^{\mathrm{T}}\mathbf{b}$$

  - ***Normal equation***
  - Has unique solution (if A^TA has full rank), which gives desired minimal mean square error

- **Essentially similar for angulation as well**

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Overview

- **Basic approaches**
- **Trilateration**
- *Multihop schemes*

Multihop range estimation

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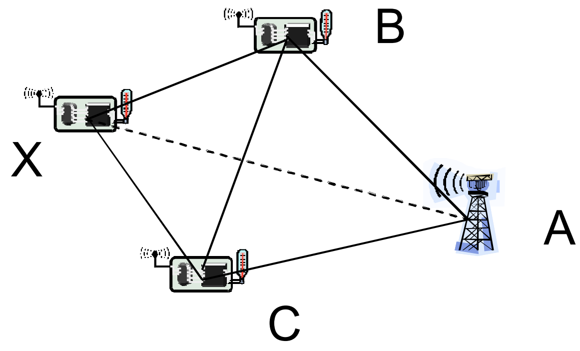
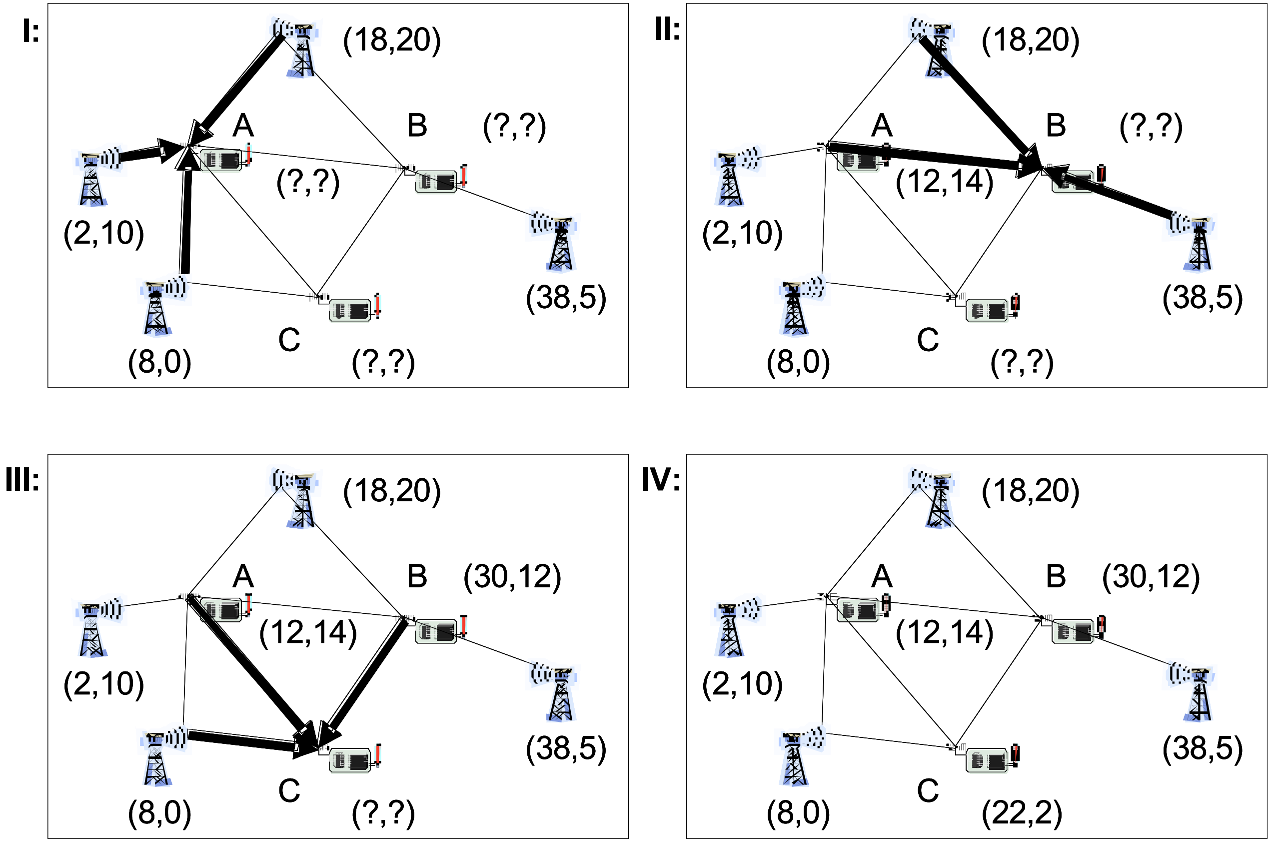
- **How to estimate range to a node to which no direct radio communication exists?**
  - No RSSI, TDoA, …
  - But: Multihop communication is possible



- **Idea 1: Count number of hops**
  - assume length of one hop is known (***DV-Hop, Niculescu et al.***)
  - Start by counting hops between anchors, divide known distance
- **Idea 2: If range estimates between neighbors exist**
  - use them to improve total length of route estimation in previous method (***DV-Distance***)

Iterative multilateration

- **Assume some nodes can hear at least three anchors (to perform triangulation), but not all**
- **Idea:**
  - let more and more nodes compute position estimates, spread position knowledge in the network
- **Problem:**
  - Errors accumulate



Probabilistic position description

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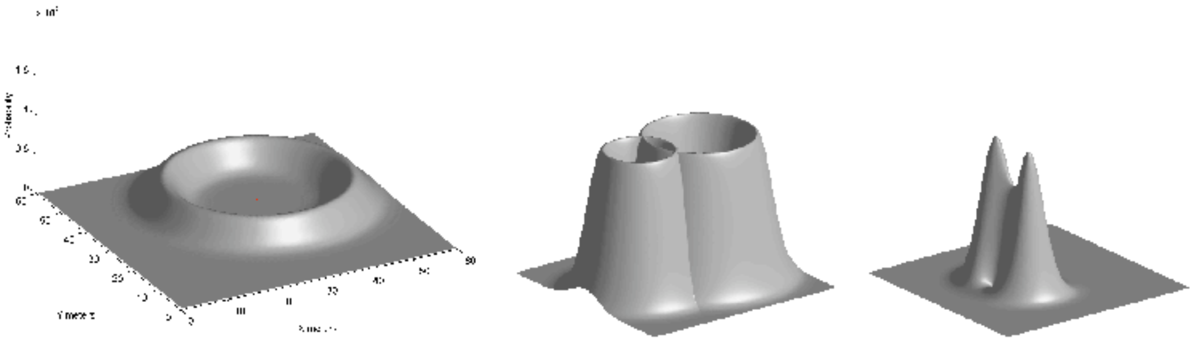
- **Similar idea to previous one, but accept problem that position of nodes is only probabilistically known**
  - Represent this probability explicitly, use it to compute probabilities for further nodes



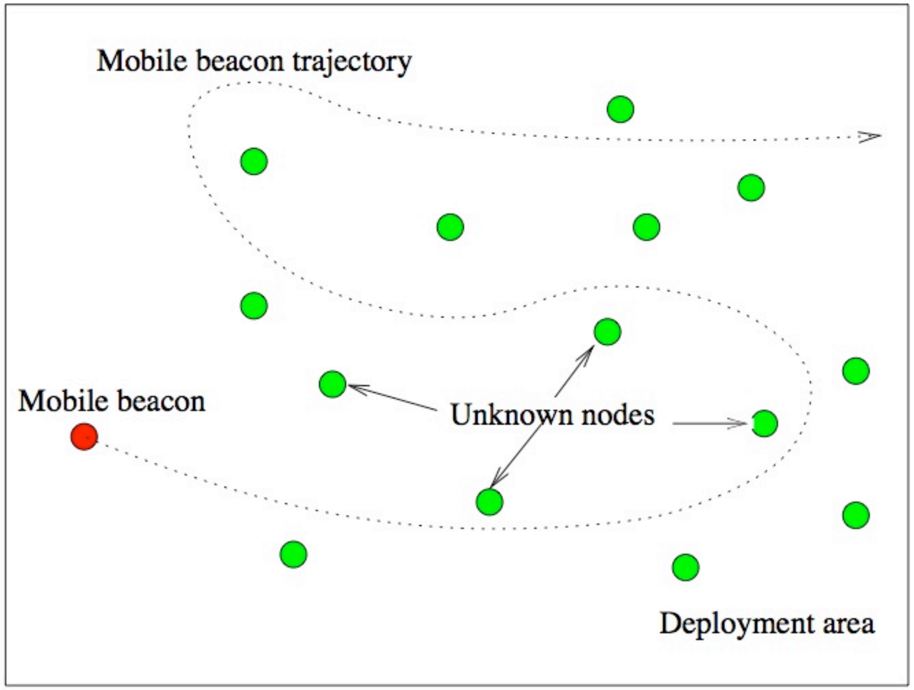
(a) Probability density function of a node positions after receiving a distance estimate from one anchor

(b) Probability density functions of two distance measurements from two independent anchors

(c) Probability density function of a node after intersecting two anchor's distance measurements

Mobile Beacon

- Localization of Wireless Sensor Networks with a Mobile Beacon
- **Mihail L. Sichitiu and Vaidyanathan Ramadurai**
  - A mobile beacon is equipped with a positioning system
  - The constraints imposed by the node being in the sensing distance of the mobile beacon.
  - Use Bayesian probability distribution analysis
  - based on RSSI (Received Signal Strength Indication)



One Measurement

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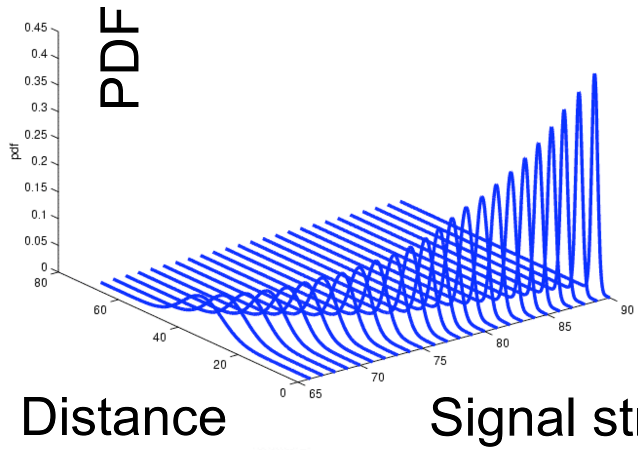
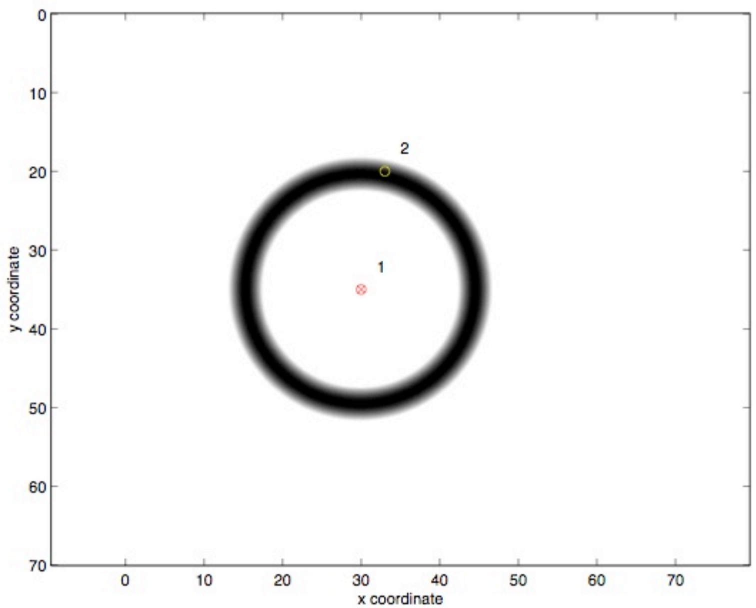
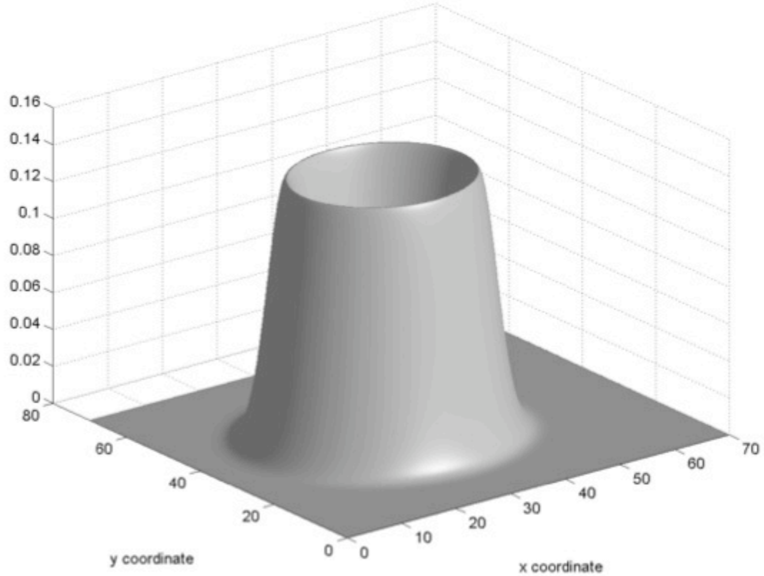
- **From one position only the following probability distribution can be derived based on RSSI**





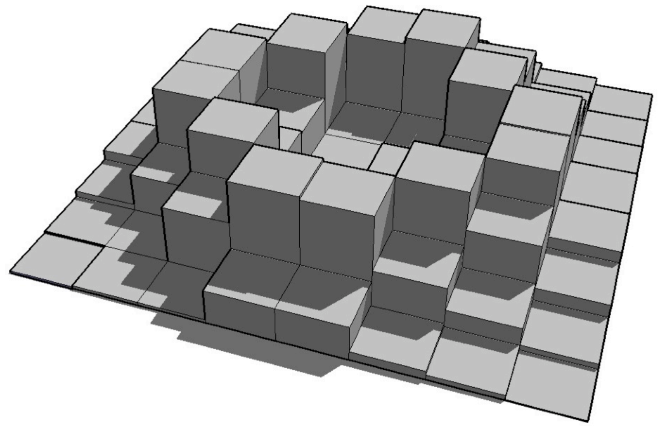
(a)



(b)

Bayes Theorem

- **Discretization**



- **Bayes Theorem:**

$$\Pr(A|B) = \frac{\Pr(B|A)\ \Pr(A)}{\Pr(B)}.$$

- **Proof**
  - By definition

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}.$$

$$\Pr(B|A) = \frac{\Pr(A \cap B)}{\Pr(A)}.$$

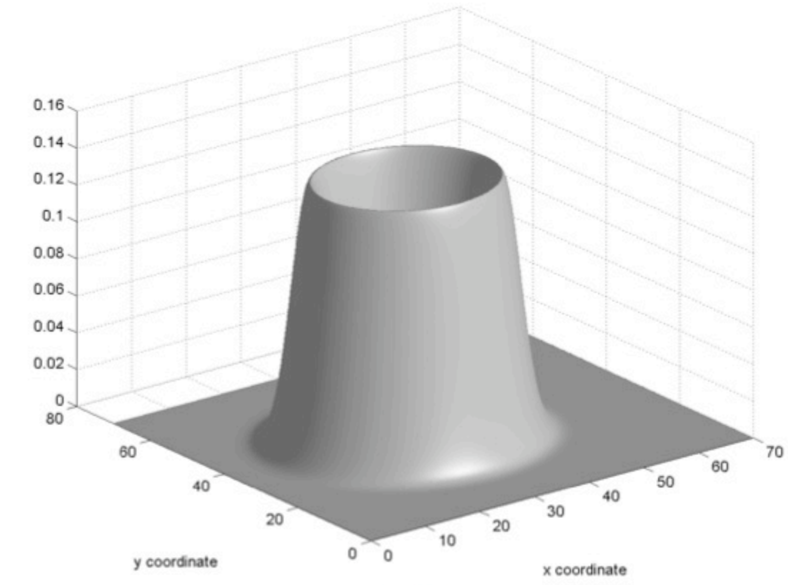
  - Combine equations

Interpretation

- A: Node is in square x
- B: RSSI is s
- Pr[A|B]: Node is in square under the condition of RSSI is s
- Pr[B|A]: RSSI is s if node is in square x

$$\Pr(A|B) = \frac{\Pr(B|A)\ \Pr(A)}{\Pr(B)}.$$

- Pr[A] is chosen according to prior measurement (or uniform)
  - Iterate estimation using multiple measurements
- Pr[B] is unknown but a constant
  - Sum over all squares to compute Pr[B]



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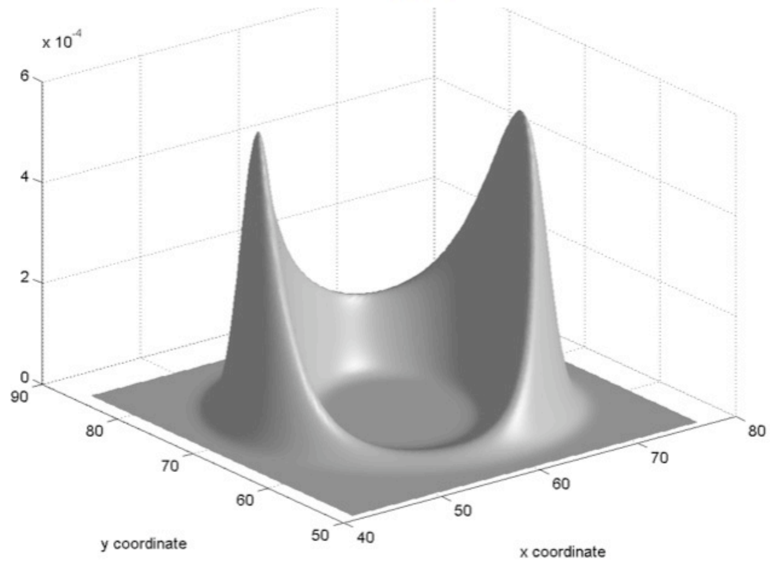

Combining Mobile Beacons with Bayes Theorem

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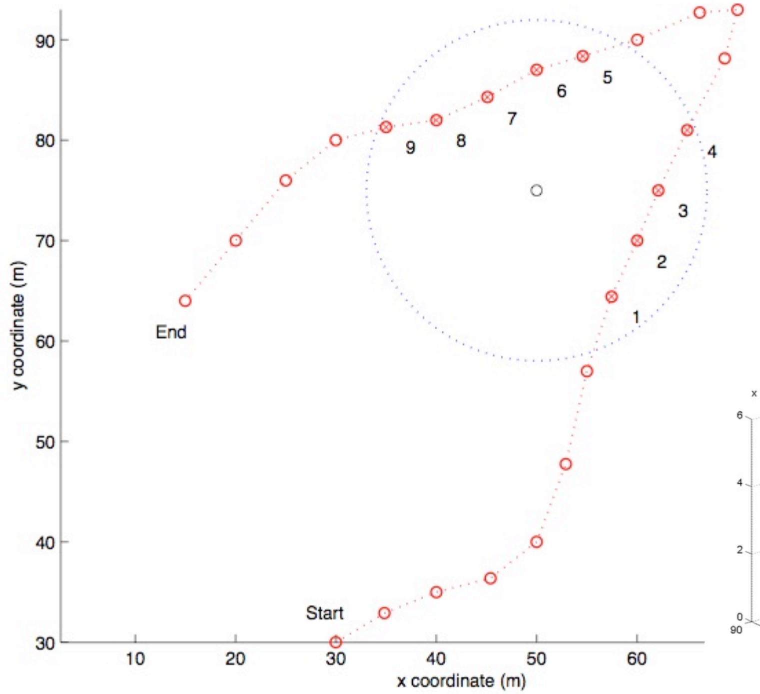

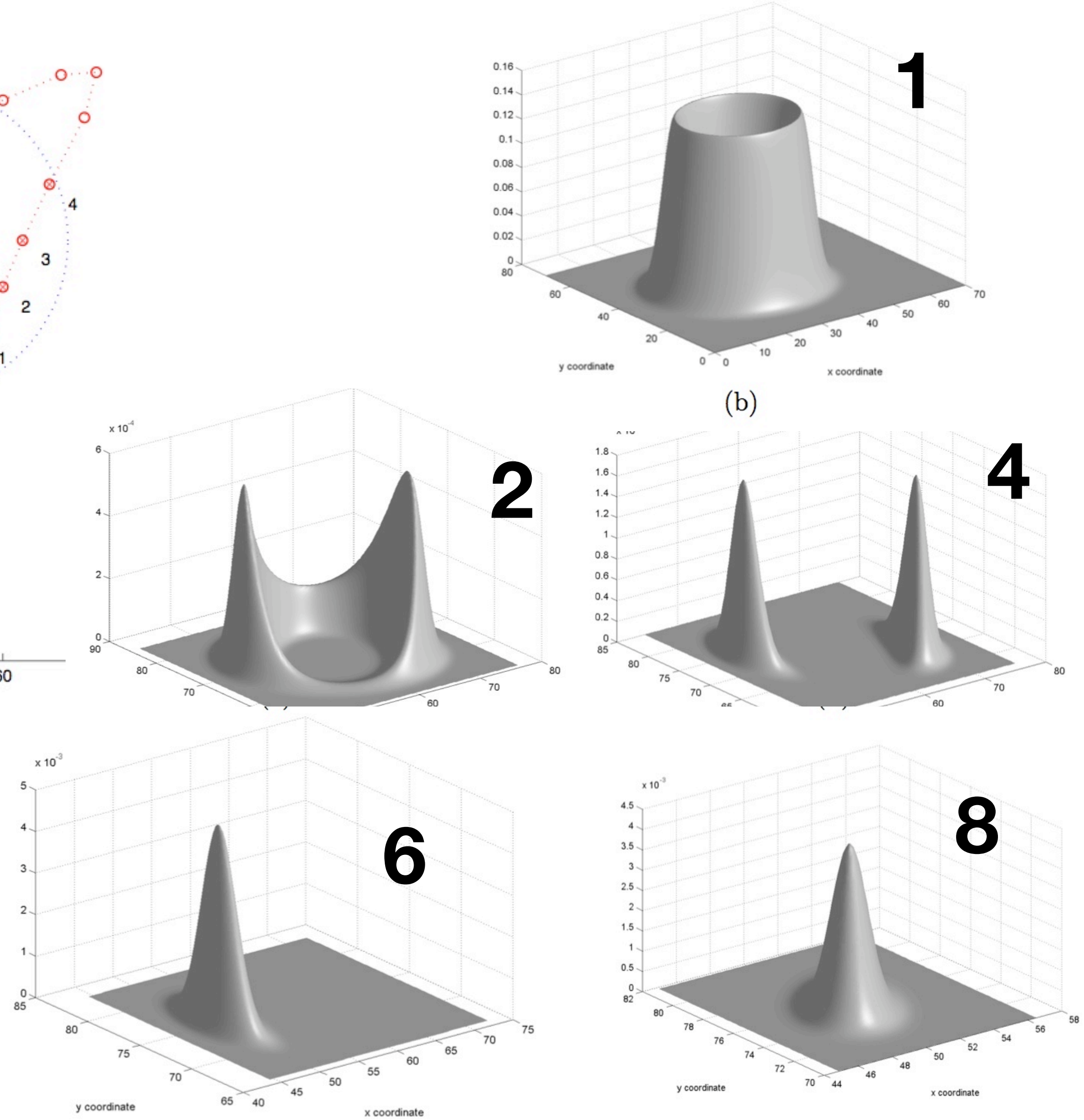

Thank you

(and thanks go also to ***Holger Karl*** *for providing some slides)*

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