Wireless Sensor Networks

19th Lecture

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CoNe Freiburg

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Options for topology control

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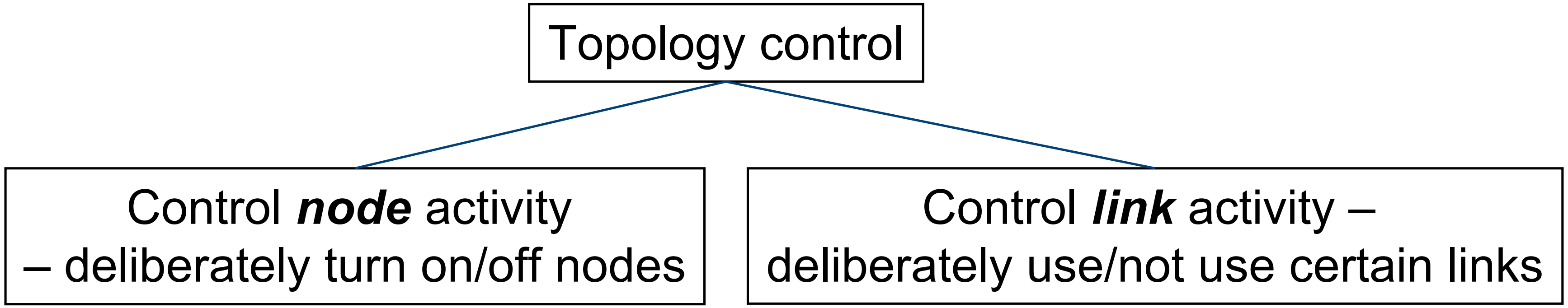

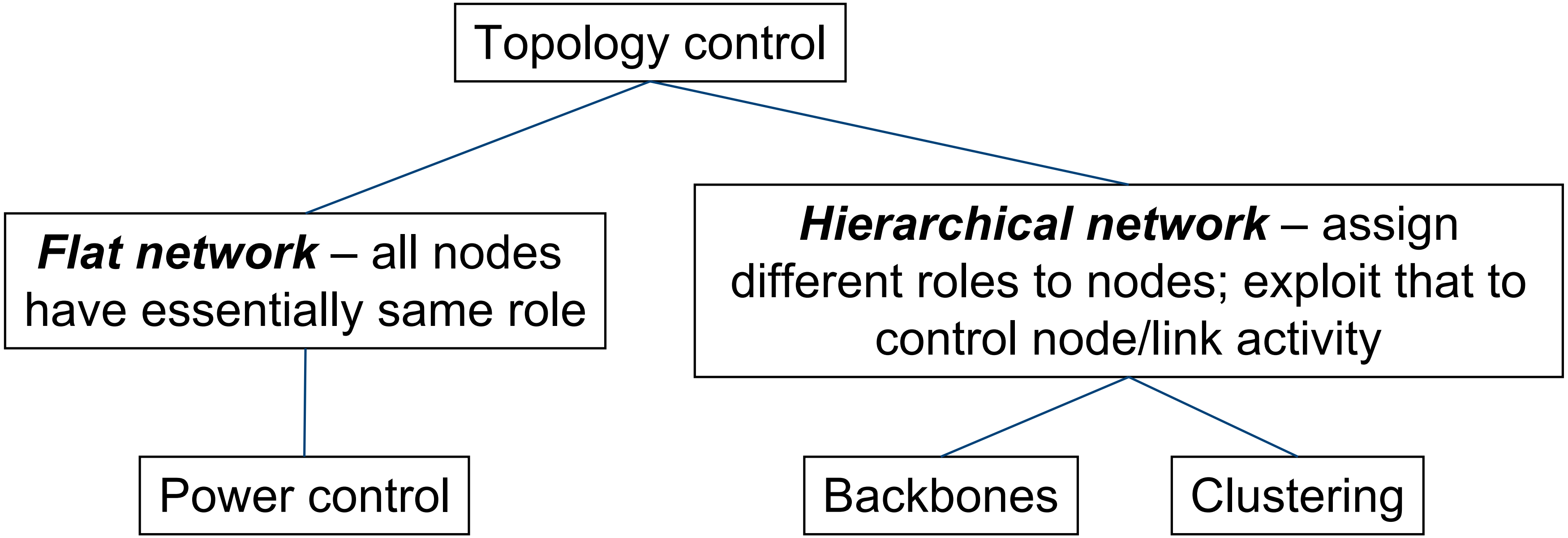

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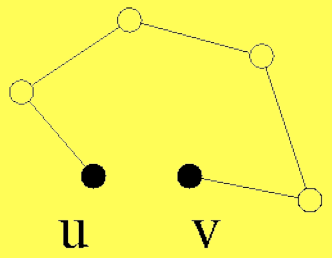
Geometric Spanner Graphs

- A Graph $G = (V, E)$ with $V \subseteq \mathbf{R}$ where for all $u, v \in V$ there exists a path $P = (u = u_1, u_2, \ldots, u_\ell = v)$ with

limited length:

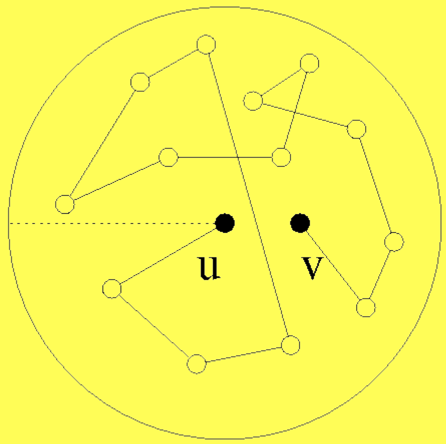
$$||P|| := \sum_{i=2}^{\ell} |u_i - u_{i-1}| \leq c \cdot |u - v|$$



c-Spanner Graph

-in a limited **radius:**

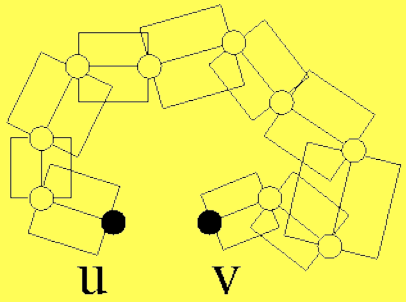
$$\max_{i=1,\ldots,\ell} |u - u_i| \leq c \cdot |u - v|$$



weak c-Spanner Graph

limited **energy costs:**

$$||P||^\delta := \sum_{i=2}^{\ell} |u_i - u_{i-1}|^\delta \leq c \cdot |u - v|^\delta$$



(c,δ)-Power-Spanner Graph

Geometric Spanners with Applications in Wireless Networks

1. Introduction
   - Definition of Geometric Spanners
   - Motivation
   - Related Work
2. Spanners versus Weak Spanners
3. Spanners versus Power Spanners
4. Weak Spanners versus Power Spanners
   - Weak Spanners are Power Spanners if Exponent > Dimension
   - Weak Spanners are not always Power Spanners if Exponent < Dimension
   - Weak Spanners are Power Spanners if Exponent = Dimension
5. Applications in Wireless Networks
6. Conclusions

Spanners versus Weak Spanners

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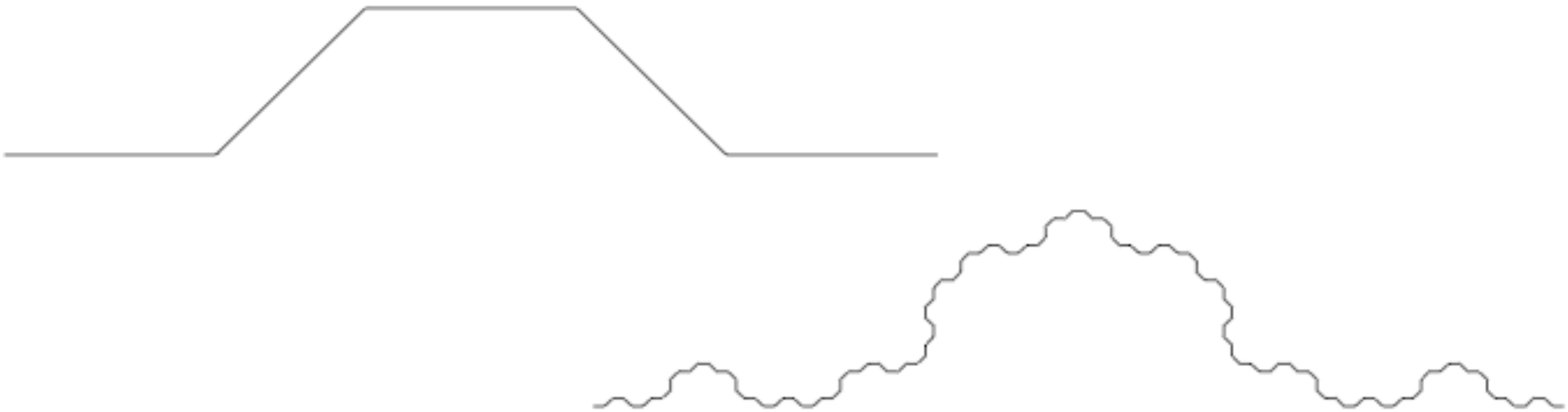
- **Fact**
  - **Every c-Spanner is also a c-Weak Spanner**
- **Theorem**
  - **There are Weak Spanner which are no Spanners**
- **Proof Idea [Eppstein]: use fractal construction**



Geometric Spanners with Applications in Wireless Networks

Spanners versus Power Spanners

- **Theorem**
  – For $\delta > 1$, every c-Spanner is also a (c^δ, δ)-Power Spanner
- **Proof:**
  – Consider a path P in the c-spanner with stretch factor c
  – This path is already a δ-Power Spanner graph with stretch factor c^δ:

$$||P||^\delta = \sum_{i=1}^{\ell-1} ||P_i||^\delta \leq \sum_{i=1}^{\ell-1} (c \cdot |u_i - u_{i+1}|)^\delta$$

$$= c^\delta \cdot \sum_{i=1}^{\ell-1} (|u_i - u_{i+1}|)^\delta = c^\delta \cdot ||P_{\mathtt{OPT}}||^\delta$$

(Weak) Spanners versus Power Spanners

- **Theorem**
  - For $\delta>1$ there is a graph family of (c,δ)-Power Spanners which are no weak C-Spanners for any constant C.
- **Proof:**
  - ...

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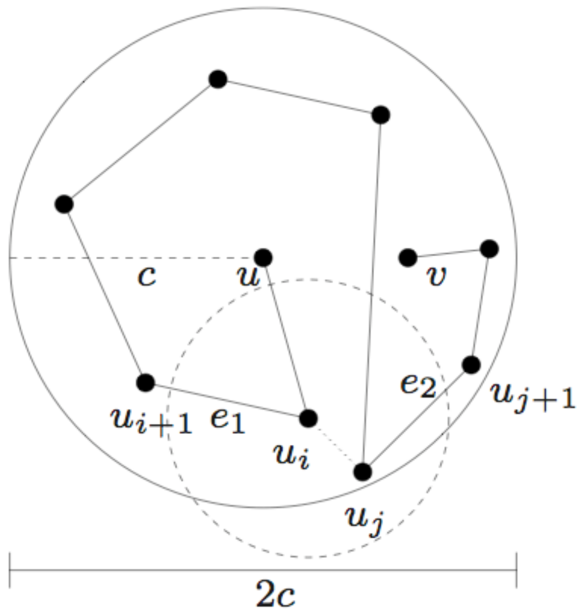
Weak Spanners are Power Spanners if Exponent > Dimension (I)

- **Lemma**
  - Let G be a weak c-spanner. Then, there is a path from nodes u to v in this graph G which as a subgraph of G is a weak 2c-spanner.
- **Proof sketch**
  - Wlog. let |u-v| = 1
  - Start with a weak c-spanner path from u to v
  - If two nodes x,y in this path are closer than 1/2
    - and the interior points of the sub-path (x,..,y) are outside the disk with center x and radius c/2
    - then construct the weak c-spanner path P' from x to y
    - and substitute the sub-path (x,..,y) with this sub-path P'
  - Repeat this process for nodes with distance 1/4, 1/8,...
  - The new path is then within a circle of radius 2c with center u



Weak Spanners are Power Spanners if Exponent > Dimension (II)

- **Lemma**

  Let $P = (u_1, \ldots, u_\ell)$ be a weak 2c-spanner, $u_i \in \mathbb{R}^D$, $|u_1 - u_\ell| = 1$. Then P contains at most $(8c+1)^D$ edges of length greater than c; more generally, P contains at most $(8c+1)^D(2^D)^k$ edges of length greater than $c/2^k$.
- **Proof sketch**
  - Consider the D-dimensional spheres of radius c
  - If the path is a weak spanner then for all pairs u,v of the path nodes the weak spanner property is valid, hence |u,v|>1/2
  - How many nodes can be gathered in the sphere of radius 1/2?
  - Consider spheres of radius 1/4. These spheres do not intersect.
  - For an upper bound divide the volume of the radius c-sphere by the volumes of the small spheres
- **Analogous for shorter edges**

Weak Spanners are Power Spanners if Exponent > Dimension (III)

- **Theorem**
  - For δ>D let G = (V , E) be a weak c-spanner with V ⊂ $\mathbf{R}^D$. Then G is a (C, δ)-power spanner for

$$C := (8c+1)^{\mathcal{D}} \cdot \frac{(2c)^\delta}{1-2^{\mathcal{D}-\delta}}$$

- **Proof sketch:**
  - Choose edge lengths from $[2^i,2^{i+1}]$
  - Sum over the edge lengths up to length c |u,v| and use:

more generally, P contains at most $(8c+1)^D(2^D)^k$ edges of length greater than $c/2^k$.

  - This leads to a converging sum for δ<D

Weak Spanners are not Power Spanners if Exponent < Dimension

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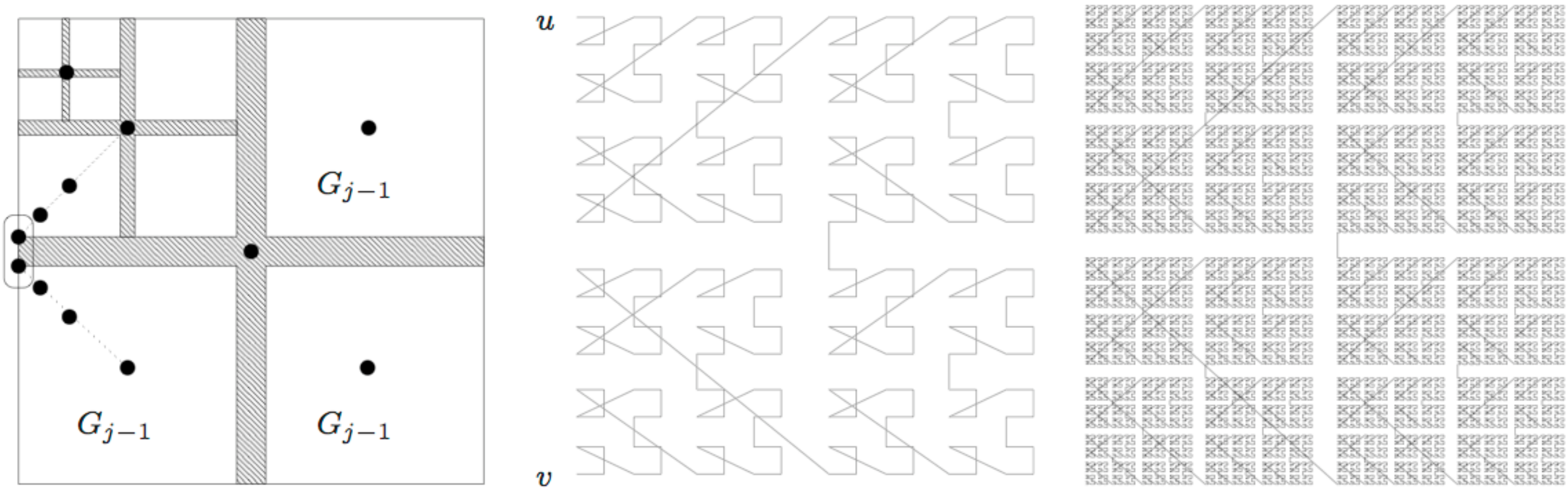
- **Theorem**
  - To any $\delta < D$, there exists a family of geometric graphs $G = (V, E)$ with $V \subset \mathbf{R}^D$ which
    - are weak c-spanners for a constant c
    - but not (C, δ)-power spanners for any fixed C .



(a) Idea (b) After 4 steps (c) After 7 steps

Weak Spanners are Power Spanners if Exponent = Dimension

- **Theorem**
  - Let G = (V , E) be a weak c-spanner with $V \subset \mathbf{R}^D$. Then G is a (C, D)-power spanner for $C := O(c^{4D})$.
- **Proof strategy:**
  1. For a two nodes (u,v) with |u,v|=1
     - Consider a bounding square of side length 4c
  2. For k = 0,1,2,..
     - Consider edges of lengths $[c\ \beta^{-k-1}, c\ \beta^{-k}[$ for some constant $\beta>1$
  3. In each iteration use “clean-up” to produce empty space
  4. If long edges exist, then at least one empty square of volume $\Omega(\beta^{-Dk})$ exist.

$$\sum_{\text{edges in round k}} \begin{pmatrix}\text{edge lengths}\\ \text{in round k}\end{pmatrix}^D = \mathcal{O}\begin{pmatrix}\text{Volume of}\\ \text{empty space}\\ \text{added in}\\ \text{round } k\end{pmatrix}$$

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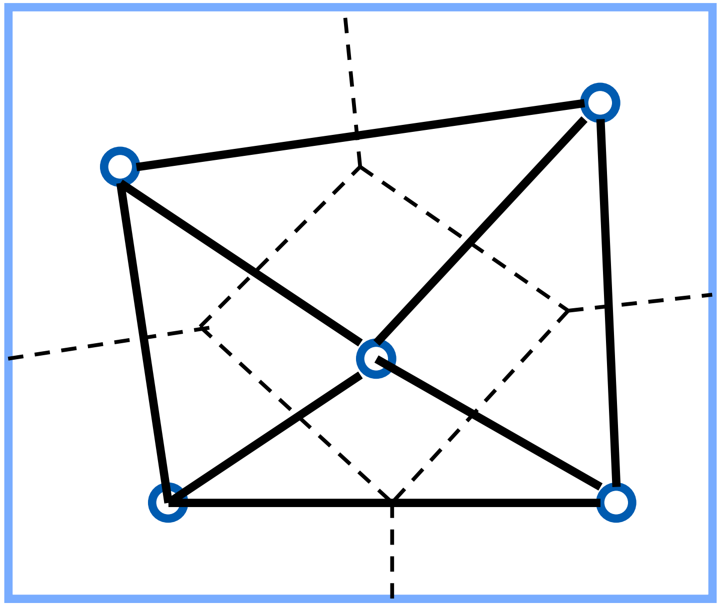
Conclusions

- **Complete characterization of the relationships of Spanners, Weak Spanners and Power Spanners**

c-**spanner**	c	c	(c^δ, δ)
weak c-**spanner**	(unbounded)	c	$\left(\mathcal{O}\left(c^{2\mathcal{D}+\epsilon}/(1-2^{-\epsilon})\right), \mathcal{D}+\epsilon\right)$ $\left(\mathcal{O}(c^{4\mathcal{D}}), \mathcal{D}\right)$ $\left(\text{unbounded}, \mathcal{D}-\epsilon\right)$
(c, δ)-**power spanner**	(unbounded)	(unbounded)	for $\Delta > \delta$: $(c^{\Delta/\delta}, \Delta)$ for $\Delta < \delta$: (unbounded, Δ)
is a	**-spanner**	**-weak spanner**	**-power spanner**

Delaunay Graph



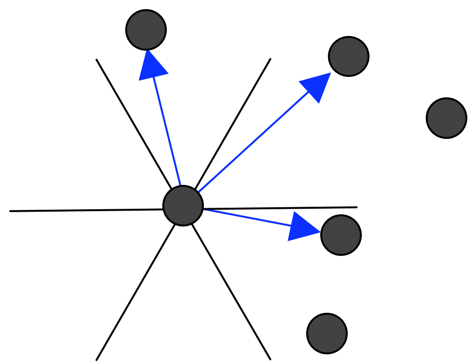
- **Definition**
  - Triangularization of a point set p such that no point is inside the circumcircle of any triangle
- **Facts**
  - Dual graph of the Voronoi-diagram
  - In 2-D
    - edge flipping leads to the Delaunay-graph
      - Flip edge if circumcircle condition is not fulfilled
    - planar graphs
    - 5.08-spanner graph
  - Problem: Might produce very long links

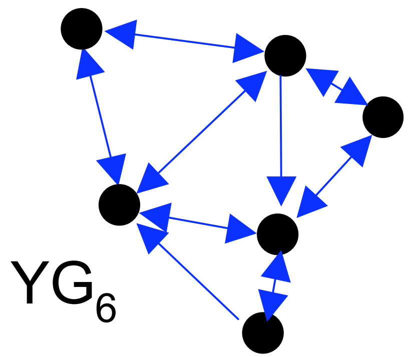
Yao-Graph

- **Choose nearest neighbor in each sector**
- **c-spanner,**
  - with stretch factor

    $$1/(1-2\sin(\theta/2))$$

  - Simple distributed construction
  - High (in-) degree





Thank you

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