Wireless Sensor Networks

21st Lecture

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Options for topology control

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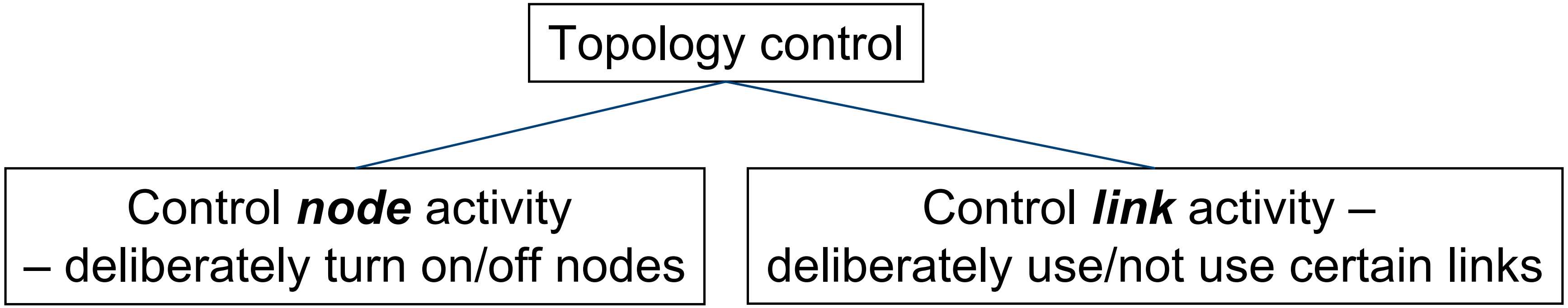

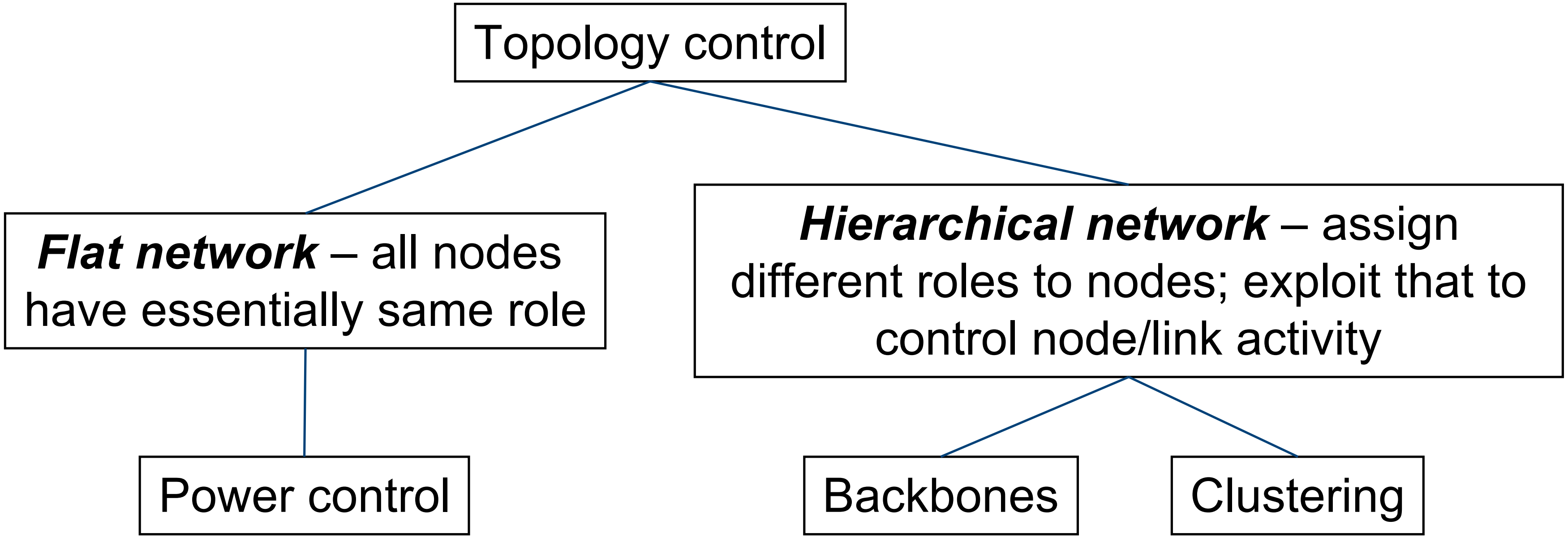

Geometric Spanner Graphs

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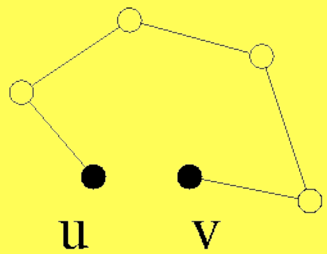
- A Graph $G = (V, E)$ with $V \subseteq \mathbf{R}$ where for all $u, v \in V$ there exists a path $P = (u = u_1, u_2, \ldots, u_\ell = v)$ with

limited length:

$$||P|| := \sum_{i=2}^{\ell} |u_i - u_{i-1}|$$

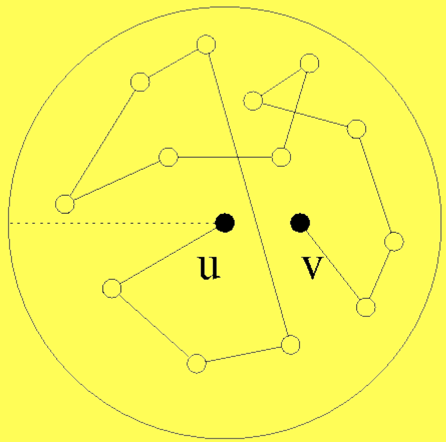
$$\leq c \cdot |u - v|$$



c-Spanner Graph

-in a limited radius:

$$\max_{i=1,\ldots,\ell} |u - u_i| \leq c \cdot |u - v|$$

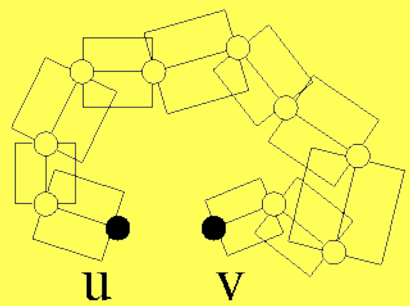


weak c-Spanner Graph

limited energy costs:

$$||P||^\delta := \sum_{i=2}^{\ell} |u_i - u_{i-1}|^\delta$$

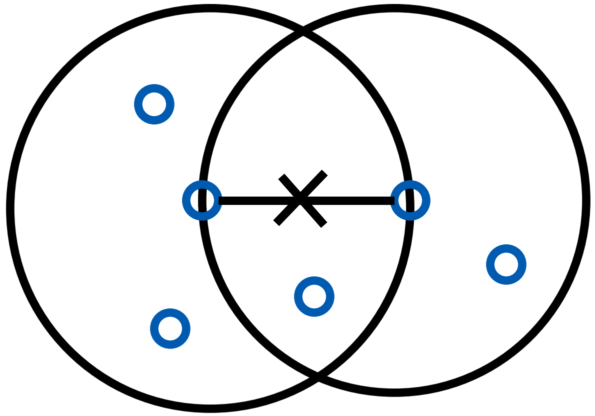
$$\leq c \cdot |u - v|^\delta$$

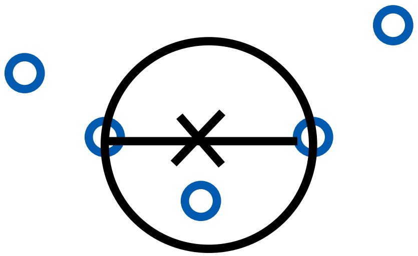


(c,δ)-Power-Spanner Graph

Proximity Graphs

- **Relative Neighborhood Graph (RNG):**
  - There is an edge between u and v only if there is no vertex w such that d(u,w) and d(v,w) are both less than d(u,v)
  - No spanners
  - Small degree





- **Gabriel Graph (GG):**
  - There is an edge between u and v if there is no vertex w in the circle with diameter chord (u,v)
  - Perfect (1,2)-power spanners
  - No strong spanners
  - Possibly high degree

Useful Structures for Multi-hop Networks

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- **Global structures:**
  - Minimum spanning trees & minimum broadcast trees
- **Local structures:**
  - Dominating sets: distributed algorithms and tradeoffs
- **Hierarchical structures:**
  - Sparse neighborhood covers

Applications of Spanning Trees

- **Forms a backbone for routing**
- **Forms the basis for certain network partitioning techniques**
- **Subtrees of a spanning tree may be useful during the construction of local structures**
- **Provides a communication framework for global computation and broadcasts**

Arbitrary Spanning Trees

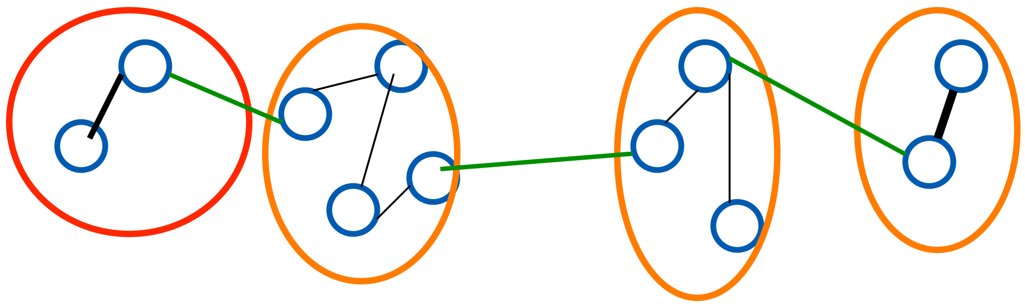
- **Trivial algorithm:**
  - A designated node starts the “flooding” process
  - When a node receives a message, it forwards it to its neighbors the first time
  - Maintain sequence numbers to differentiate between different computations
- **Properties**
  - Nodes can operate asynchronously
  - Number of messages is $O(m)$;worst-case time,
  - for synchronous control, is $O(\mathrm{Diam}(G))$

Minimum Spanning Trees

- **Centralized algorithms**
  - Prim's algorithm [57]
    - Increase the connected component by adding the lightest adjacent edge not in the component
  - Kruskal's algorithm [56]
    - Add small weight edges (leaving out edges within a connected component)
- **The basic algorithm [Gallagher-Humblet-Spira 83]**
  - $O(m + n \log n)$ messages and $O(n \log n)$ time
- **Improved time and/or message complexity [Chin-Ting 85, Gafni 86, Awerbuch 87]**
- **First sub-linear time algorithm [Garay-Kutten-Peleg 93]:**
  - Improved to $O(\mathrm{D} + n^{0.61} \log^* n)$
- **Taxonomy and experimental analysis**
  - [Faloutsos-Molle 96] $O(D + \sqrt{n} \log^* n)$
  - lower bound [Rabinovich-Peleg 00] $\Omega(D + \sqrt{n} / \log n)$

The Basic Algorithm

- **Distributed implementation of Borouvka's algorithm [Borouvka 26]**
- **Each node is initially a fragment**
- **Fragment F_1 repeatedly finds a min-weight edge leaving it and attempts to merge with the neighboring fragment, say**
  - If fragment F_2 also chooses the same edge, then merge F_2
  - Otherwise, we have a sequence of fragments, which together form a fragment



Subtleties in the Basic Algorithm

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- **All nodes operate asynchronously**
- **When two fragments are merged, we should "relabel" the smaller fragment.**
- **Maintain a level for each fragment and ensure that fragment with smaller level is relabeled:**
  - When fragments of same level merge, level increases; otherwise, level equals larger of the two levels
- **Inefficiency: A large fragment of small level may merge with many small fragments of larger levels**

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Asymptotic Improvements to the Basic Algorithm

- **The fragment level is set to log of the fragment size [Chin-Ting 85, Gafni 85]**
  - Reduces running time to $O(n \log^* n)$
- **Improved by ensuring that computation in level fragment is blocked for time $O(2^\ell)$**
  - Reduces running time to $O(n)$

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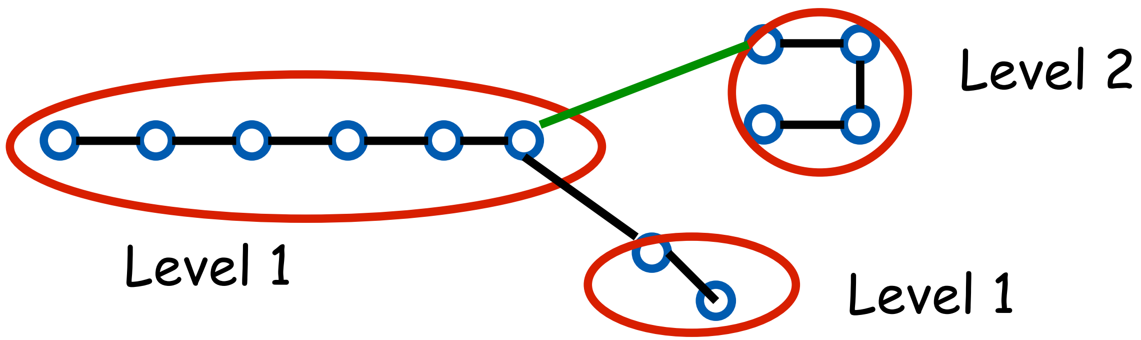

A Sublinear Time Distributed Algorithm

- **All previous algorithms perform computation over fragments of MST, which may have diameter $\Omega(n)$**
- **Two phase approach [GKP 93, KP 98]**
  - Controlled execution of the basic algorithm, stopping when fragment diameter reaches a certain size
  - Execute an edge elimination process that requires processing at the central node of a BFS tree
- **Running time is $O(\mathrm{Diam}(G) + \sqrt{n} \log^* n)$**
- **Requires a fair amount of synchronization**

Minimum Energy Broadcast Routing

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- **Given a set of nodes in the plane, need to broadcast from a source to other nodes**
- **In a single step, a node may broadcast within a range by appropriately adjusting transmit power**
- **Energy consumed by a broadcast over range** r **is proportional to**
- **Problem: Compute the sequence of broadcast steps that consume minimum total energy** r^{α}
  - Optimum structure is a directed tree rooted at the source

Energy-Efficient Broadcast Trees

- **NP-hard for general graphs, complexity for the plane still open**
- **Greedy heuristics proposed [Wieselthier et al 00]**
  - MST: Minimum spanning tree with edge weights equal to energy required to transmit over the edge
  - SPT: Shortest path tree with same weights
  - BIP: Bounded Incremental Power: Add next node into broadcast tree, that requires minimum extra power
- **MST and BIP have constant-factor approximation ratios**
- **SPT (Shortest-Path Tree) has ratio $\Omega(n)$ [Wan et al 01]**
  - If weights are square of Euclidean distances, then MST for any point set in unit disk is at most 12

Thank you

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