# Fast Message Dissemination in Random Geometric Ad-Hoc Radio Networks*

Artur Czumaj[1] and Xin Wang[2]

[1] Department of Computer Science, University of Warwick, Coventry CV4 7AL, United Kingdom
czumaj@dcs.warwick.ac.uk

[2] Department of Computer Science, New Jersey Institute of Technology, Newark, NJ 07102-1982, USA
xw37@oak.njit.edu

**Abstract.** We study the complexity of distributed protocols for the classical information dissemination problem of *distributed gossiping*. We consider the model of *random geometric networks*, one of the main models used to study properties of sensor and ad-hoc networks, where $n$ points are randomly placed in a unit square and two points are connected by an edge/link if they are at at most a certain fixed distance $r$ from each other. To study communication in the network, we consider the *ad-hoc radio networks* model of communication. We examine various scenarios depending on the local knowledge of each node in the networks, and show that in many settings distributed gossiping in asymptotically optimal time $\mathcal{O}(D)$ is possible, where $D$ is the diameter of the network and thus a trivial lower bound for any communication.

## 1 Introduction

In this paper we study basic communication properties of random geometric networks as motivated by *mobile ad hoc networks* and *sensor networks*. Our main goal is to study under what conditions the dissemination of information can be performed efficiently, in particular, in time proportional to the diameter of the underlying network. We concentrate on the classical communication problem of *gossiping*: disseminating the messages in a network so that each node will receive messages from all other nodes.

*Network model.* We consider the standard model of *random geometric networks* [21]. A random geometric network $\mathcal{N} = (V, E)$ is an undirected graph with node set $V$ corresponding to the set of transmitter-receiver stations placed *independently and uniformly at random* (i.u.r.) [1] in the unit square $[0, 1]^2$. The edges $E$ of $\mathcal{N}$ connect specific pairs of nodes. We consider the *unit disc graph* model in

* Research supported in part by the *Centre for Discrete Mathematics and its Applications (DIMAP)*, University of Warwick.

[1] Another classical model assumes the points with *Poisson distribution* in $[0, 1]^2$. All our analyzes will work in that model too.

T. Tokuyama (Ed.): ISAAC 2007, LNCS 4835, pp. 220–231, 2007.

which for a given parameter $r$ (called the *radius*) there is an edge between two nodes $p, q \in V$ if and only if the distance between $p$ and $q$ (denoted by $\mathsf{dist}(p, q)$) is smaller than or equal to $r$.

To study communication in the network, we consider the so-called *ad-hoc radio networks* model of communication [1,6,7,9,10,11,18]. We assume that all nodes have access to a global clock and work synchronously in discrete time steps called *rounds*. In radio networks the nodes communicate by sending messages through the edges of the network. In each round each node can either transmit the message to all its neighbors at once or can receive the message from one of its neighbors (be in the listening mode). A node $x$ will receive a message from its neighbor $y$ in a given round if and only if it does not transmit (is in the listening mode) and $y$ is the only neighbor of $x$ that is transmitting in that round. If more than one neighbor transmits simultaneously in a given round, then a collision occurs and no message is received by the node. In that case, we assume that the node cannot distinguish such a collision from the situation when none of its neighbors is transmitting. Furthermore, we assume the length of the message sending in one round is polynomial of $n$, and thus, each node can combine multiple messages into one.

*Geometric models of knowledge.* We consider the model of *ad-hoc networks*, in which the topology of the connections is not known in advance. In general, the nodes do not know their positions nor they know the positions of their neighbors, and each node only knows its ID (a unique integer in $[1, n^{\lambda}]$ for an arbitrary constant $\lambda$; this assumption can be removed in the randomized algorithm), its initial message, and the number of the nodes $n$ in $\mathcal{N}$. (Since in all our settings, the running time is polynomial in $n$ (because $D$ is polynomial in $n$), this assumption can be removed by the standard doubling technique, without change the asymptotic time complexity.)

In many applications, one can assume that the nodes of the network have some additional devices that allow them to obtain some basic (geometric) information about the network. The most powerful model assumes that each node has a low-power Global Position System (GPS) device, which gives the node its exact location in the system [13]. Since GPS devices are relatively expensive, GPS is often not available. In such situation, we consider a *range-aware* model, the model extensively studied in the context of localization problem for sensor networks [2]. In this model, the distance between neighboring nodes is either known or can be estimated by received signal strength (RSS) readings with some errors. We also consider another scenario, in which each node can be aware of the direction of the incoming signals, that is, to measure the angles between different neighbors [20].

*Properties of random geometric networks.* It is known that when $r < (1 - o(1)) \cdot \sqrt{\ln n/(\pi n)}$, the network is disconnected with high probability [14], and therefore gossiping is meaningless in that case. Therefore, in this paper we will always assume that $r \geq c \cdot \sqrt{\log n/n}$ for some sufficiently large constant $c$. This ensures that the network is connected with high probability and therefore gossiping is feasible. With this assumption we can also make some further assumptions about the structure of the input network. And so, it is well known (cf. [21]) that such a random geometric network has *diameter* $D = \Theta(1/r)$ and the minimum and maximum degree is $\Theta(n\,r^2)$, where all these claims hold *with high probability*, that is, with probability at least $1 - 1/n^{\Omega(1)}$. Therefore, from now on, we shall *implicitly* condition on these events.

*Related prior works.* In the *centralized scenario*, when each node knows the entire network, Kowalski and Pelc [18] gave a centralized deterministic broadcasting algorithm running in $\mathcal{O}(D + \log^2 n)$ time and Gąsieniec et al. [11] designed a deterministic $\mathcal{O}(D + \Delta \log n)$-time gossiping algorithm, where $D$ is the diameter and $\Delta$ the maximum degree of the network.

There has been also a very extensive research in the *non-centralized (distributed)* setting in ad-hoc radio networks, see, e.g., [3,7,12,17,18] and the references therein. In the model of *unknown topology networks*, randomized broadcasting can be performed in the optimal $\mathcal{O}(D \log(n/D) + \log^2 n)$ time [7,17]; fastest deterministic algorithm runs in $\mathcal{O}(n \log^2 D)$ time [7]. The fastest randomized algorithm for gossiping in directed networks runs in $\mathcal{O}(n \log^2 n)$ time [7]; fastest deterministic one runs in $\mathcal{O}(n^{4/3} \log^4 n)$ time [12]. For undirected networks, both broadcasting and gossiping have deterministic $\mathcal{O}(n)$-time algorithms [1,4].

Dessmark and Pelc [9] consider broadcasting in ad-hoc radio networks in a model of geometric networks. They consider scenarios in which all nodes either *know their own locations in the plane*, or the labels of the nodes within some distance from them. The nodes use disks of possibly different sizes to define their neighbors. Dessmark and Pelc [9] show that *broadcasting* can be performed in $\mathcal{O}(D)$ time.

Recently, the complexity of broadcasting in ad-hoc radio networks has been investigated in a *(non-geometric)* model of $G_{n,p}$ *random networks* by Elsässer and Gąsieniec [10], and Chlebus et al. [5], and in the model of random line-of-sight ad-hoc radio networks by Czumaj and Wang [8].

*New contributions.* In this paper we present a thorough study of basic communication primitives in *random geometric ad-hoc radio networks.* We study information dissemination in various models of random geometric ad-hoc radio networks and we demonstrate that in many scenarios, the random structure of these networks allows us to perform distributed gossiping in asymptotically optimal time $\mathcal{O}(D)$.

We begin with the most restrictive model of local knowledge, the *unknown topology model.* In this model, the nodes have no global nor local information about the structure of the network. Still, we show that it is possible to perform distributed randomized gossiping in $\mathcal{O}(n\,r^2 \log n + D)$ time, with high probability. This is the first asymptotically optimal algorithm for random geometric ad-hoc radio unknown topology networks with $r \le \mathcal{O}((n \log n)^{-1/3})$, in which case the running time is $\mathcal{O}(D)$.

Next, we consider deterministic distributed algorithms in three models in which the nodes have some geometric local information about the network. In the first model we consider, if a node communicates with another node, then he is able to determine the *distance* to the node with which he communicates. In the next model, each node is able to determine *directions* to all its neighbors. Finally we consider the most powerful model in which each node knows its own *position* in $[0,1]^2$. The first two models are fairly similar and for them we design a distributed deterministic algorithms complete gossiping in optimal $\mathcal{O}(D)$ time, assuming $r \leq \mathcal{O}(n^{-7/16} \log^{-5/16} n)$. The model in which each node knows its own location is more powerful and we use the techniques from [9] to get a $\mathcal{O}(D)$-time deterministic algorithm for even larger range of $r$, $r \leq \mathcal{O}(n^{-2/5} \log^{-1/5} n)$.

For majority of applications of random geometric ad-hoc radio networks the underlying networks are sparse or are aimed to be as sparse as possible. Therefore, even though we present our algorithms to work for all values of $r \geq c\sqrt{\log n/n}$, our main focus is on networks with small values of $r$, just a little above connectivity threshold. For such networks, our algorithms have asymptotically optimal running times for a large range of the parameter $r$.

## 2 Preliminaries

For any node $v$, define $N(v)$ to be the set of nodes that are reachable from $v$ in one hop, $N(v) = \{u \in V : \mathsf{dist}(v,u) \leq r\}$, where $\mathsf{dist}(v,u)$ is the Euclidean distance between $v$ and $u$. Any node in $N(v)$ is called a *neighbor* of $v$, and set $N(v)$ is called the *neighboring set* of $v$. For any $X \subseteq V$, let $N(X) = \bigcup_{x \in X} N(x)$. Define the *kth neighborhood* of a node $v$, $N^k(v)$, recursively as follows: $N^0(v) = v$ and $N^k(v) = N(N^{k-1}(v))$ for $k \geq 1$. The *strict kth neighborhood* of $v$, denoted by $SN^k(v)$, is defined as $SN^k(v) = N^k(v) \setminus N^{k-1}(v)$.

*Strongly-selective families.* Let $k$ and $m$ be two arbitrary positive integers with $k \leq m$. Following [3], a family $\mathcal{F}$ of subsets of $\{1,\ldots,m\}$ is called $(m,k)$-*strongly-selective* if for every subset $X \subseteq \{1,\ldots,m\}$ with $|X| \leq k$, for every $x \in X$ there exists a set $F \in \mathcal{F}$ such that $X \cap F = \{x\}$. It is known (see, e.g., [3]) that for every $k$ and $m$, there exists a $(m,k)$-strongly-selective family of size $\mathcal{O}(k^2 \log m)$.

With the concept of strongly-selective families, we are now ready to proceed to the following lemma.

**Lemma 1.** *In random geometric networks, for any integer $k$, in **(deterministic)** time $\mathcal{O}(k \cdot n^2 \cdot r^4 \cdot \log n)$ all nodes can send their messages to all nodes in their kth neighborhood. The algorithm may fail with probability at most $1/n^2$ (where the probability is with respect to the random choice of a geometric network).*

*Proof.* The proof uses nowadays standard approach of applying selective families to broadcasting and gossiping in radio ad-hoc networks, see, e.g., [3]. □

## 3 Randomized Gossiping in Optimal $\mathcal{O}(D)$ Time

In this section, we present a simple randomized algorithm for broadcasting and gossiping problem in random geometric networks whose running time is asymptotically optimal for small values of $r$. We see our algorithm as an extension of the classical broadcasting algorithm in networks due to Bar-Yehuda et al. [1] (see also [7]), which when applied to random geometric networks gives asymptotically optimal runtime for a more complex task of gossiping (for small $r$).

> **repeat**
> **in each round, each node independently does:**
> the node transmits with probability $\frac{1}{n\,r^2}$

**Theorem 1.** *The algorithm above completes gossiping in a random geometric network after* $\mathcal{O}(n\,r^2 \log n + D)$ *rounds with probability at least* $1 - 1/n$*. If* $r \leq \mathcal{O}\left(\frac{1}{(n \log n)^{1/3}}\right)$*, then the number of rounds is* $\mathcal{O}(D)$*.*

Before we proceed with the proof of Theorem 1, let us first introduce some basic notation. Let us divide the unit square into $16/r^2$ *blocks* (disjoint squares), each block with the side length of $r/4$. For a block $B$, we also use $B$ to denote the set of nodes in block $B$; in this case, $|B|$ is the number of nodes in block $B$.

The following lemma follows easily from Chernoff bounds.

**Lemma 2.** *For every block $B$ with probability at least* $1 - 1/n^4$*: (i)* $\frac{n\,r^2}{32} \leq |B| \leq n\,r^2$*, (ii)* $|N(B)| \leq 20\,n\,r^2$*.*

A *gossiping within a block* is the task of exchanging the messages between all the nodes in the block. Gossiping within a block $B$ is completed if every node $v \in B$ receives a message from every other $u \in B$.

**Lemma 3.** *Gossiping within every block completes in* $\mathcal{O}(n\,r^2 \log n)$ *steps with probability at least* $1 - \frac{1}{n^2}$*.*

*Proof.* Fix a node $v \in B$. In any single round, the probability that node $v$ transmits and no other node from $N(B) \setminus \{v\}$ transmits is at least $\frac{1}{n\,r^2}(1 - \frac{1}{n\,r^2})^{|N(B)\setminus\{v\}|} \geq \frac{1}{n\,r^2}(1 - \frac{1}{n\,r^2})^{20\,n\,r^2} \geq \frac{1}{n\,r^2}\,e^{-40}$. Hence, in any single step, $v$ will send its message to all other nodes in block $B$ with probability at least $\frac{1}{e^{40}\,n\,r^2}$. After $\tau$ steps, $v$ sends its message to all other nodes in block $B$ with probability at least $1 - (1 - \frac{1}{e^{40}\,n\,r^2})^{\tau}$. Hence, by the union bound, the probability that the gossiping within every single block will be completed after $\tau$ steps is greater than or equal to $1 - n \cdot (1 - \frac{1}{e^{40}\,n\,r^2})^{\tau}$. By choosing an appropriate large value of $\tau = \mathcal{O}(n\,r^2 \log n)$, this probability will be greater than $1 - \frac{1}{n^2}$, as needed.

At any time step $t$, let $M_t(v)$ be the set of messages currently held by node $v$. For any block $B$, let $M_t(B)$ denote the set of common messages that are currently held by all nodes of $B$, that is, $M_t(B) = \bigcap_{v \in B} M_t(v)$.

**Lemma 4.** *Let $B$ and $B'$ be two adjacent blocks and suppose that the gossiping within block $B$ has been completed. Then, for any $t$,* $M_t(B) \cup M_t(B') \subseteq M_{t+1}(B')$ *with constant probability.*

*Proof.* By Lemma 2, $|N(B')| \le 20\, n\, r^2$ and $|B| \ge n\, r^2/32$ with high probability. Therefore, conditioned on these two inequalities, with probability $p \ge |B| \cdot \frac{1}{n\, r^2} \cdot (1 - \frac{1}{n\, r^2})^{|N(B')|} \ge n\, r^2/32 \cdot \frac{1}{n\, r^2} \cdot (1 - \frac{1}{n\, r^2})^{20\, n\, r^2}$, among all nodes in $N(B')$, there is exactly one node in $B$ that transmits at a given time step. For $n$ big enough, $p$ is greater than some positive constant $c'$. This yields the claim.

Now, we are ready to complete the proof of Theorem 1. Let us focus on two blocks $B$ and $B'$. By Lemma 3, gossiping within every block will be completed after the first $\mathcal{O}(n\, r^2 \log n)$ steps w.h.p. For fixed blocks $B$ and $B'$, there is always a sequence of blocks $B = B_1, B_2, \ldots, B_k = B'$, such that $B_i$ and $B_{i+1}$ are adjacent for any $1 \le i \le k-1$, and that $k \le 8/r$. By Lemma 4, after each step, $B_i$ will send its message $M_t(B_i)$ to $B_{i+1}$ with probability at least $c'$, where $c'$ is a positive constant promised by Lemma 4. Therefore, by a simple application of known concentration results for random variables with negative binomial distribution, after $\mathcal{O}(k/c' + \log n) = \mathcal{O}(D + \log n)$ steps, all the messages from $B$ will be successfully transmitted to $B'$ with probability at least $1 - 1/n^4$. By applying the union bound on all pairs of blocks, we conclude that gossiping is completed with probability at least $1 - 1/n^2$. $\square$

*Randomized broadcasting.* Our analysis in Theorem 1 can be improved for the broadcasting problem, where for every $r$ we can obtain the running time of $\mathcal{O}(D + \log n)$. (Details deferred to the full version.)

## 4 Deterministic Distributed Algorithm: Knowing Distances Helps

In this section, we assume that $c\sqrt{\log n/n} \le r \le \mathcal{O}(n^{-7/16} \log^{-5/16} n)$ and show that the gossiping in random geometric networks can be done optimally in time $\mathcal{O}(D)$ in the *range-aware* model.

*Building a local map.* The key property of our model that we will explore in the optimal gossiping algorithm is that by checking the inter-point distances, we can create a "map" with relative locations of the points. Indeed, if for three points $u, v, w$, we know their inter-points distances, then if we choose $u$ to be the origin (that is, has location $(0, 0)$), we can give relative locations of the other two points $v$ and $w$. (The relative location is not unique because there are two possible locations, but by symmetry, any of these two positions will suffice for our analysis.) We will show later that with such a map, the gossiping task can be performed optimally. (Let us point out that even with local coordinate system, the global consistent position information is still unavailable.)

The following lemma easily follows from Lemma 1 and the discussion above.

**Lemma 5.** *After $\mathcal{O}(D)$ communication steps, all nodes $u \in \mathcal{N}$ can learn $\mathsf{dist}(u, v)$ for any node $v \in N^{\tau}(u)$, where $\tau = \lceil 1/(n^2\, r^5 \log n) \rceil$. (This algorithm may fail with probability at most $1 - 1/n^3$.)*

This lemma implies not only that $u \in \mathcal{N}$ can learn $\mathsf{dist}(u, v)$ for any node $v \in N^\tau(u)$, but also that it can set up its own local map of the nodes in $N^\tau(u)$. From now on, we will proceed with $\tau = \lceil 1/(n^2 r^5 \log n) \rceil$.

*Boundary and corner nodes.* In our algorithm we consider two special types of nodes: *boundary nodes* and *corner nodes.*

If a node $u$ observes that there is a sector with angle $\pi/2$ that is centered at $u$ so that every neighbor of $u$ in that sector is at a distance at most $r/\sqrt{2}$, then $u$ marks itself as a *boundary node.* It is easy to see that with high probability, a node is a boundary node only if its distance to the boundary of $[0, 1]^2$ is less than $r$, and also every node which is at a distance at most $r/2$ from the boundary is a boundary node. Similarly, a node $u$ marks itself as a *corner node* if there is a line going through $u$ for which all neighbors of $u$ that are on one side of the line have distance at most $r/2$ from $u$. It is easy to see that with high probability, every corner node is at a distance at most $r$ from a corner of $[0, 1]^2$ and every node that is at a distance at most $r/4$ from a corner of $[0, 1]^2$ is a corner node.

Next, we select one *corner representative node* for each corner of $[0, 1]^2$. It can be done easily by Lemma 1.

*Transmitting along boundary nodes.* Now, we will show that the gossiping among boundary nodes can be performed in optimal $\mathcal{O}(D)$ time.

The process of the gossiping among the boundary nodes is initialized by the four corner representative nodes. Each corner representative node $u$ checks its map of the nodes in $N^\tau(u)$ and selects two farthest boundary nodes, one for each boundary. Then, it sends a message to these two nodes with the aim of transmitting its message to the two neighboring corner representative nodes

The process of sending messages to the corners works in *phases.* In each phase, there are up to eight pairs of nodes $\varpi_i^j$ and $\varpi_{i+1}^j$ such that $\varpi_i^j$ wants to transmit a message to $\varpi_{i+1}^j$, with both $\varpi_i^j$ and $\varpi_{i+1}^j$ being boundary nodes and $\varpi_{i+1}^j \in N^\tau(\varpi_i^j)$. At the beginning of the phase, $\varpi_i^j$ checks its local map and finds a path $P_{ij}$ from $\varpi_i^j$ to $\varpi_{i+1}^j$ of length at most $\tau$. Then, it transmits to its neighbors and request that only the first node on $P_{ij}$ will transmit the message to $\varpi_{i+1}^j$. Then, the first node on $P_{ij}$ will transmit to its neighbors and will request that only the second neighbor on $P_{ij}$ will transmit, and so on, until $\varpi_{i+1}^j$ will receive the message. Once $\varpi_{i+1}^j$ received a message, it sends back an acknowledgement to $\varpi_i^j$ that the message has been delivered. The algorithm for sending an acknowledgement is a reverse of the algorithm for transmitting a message from $\varpi_i^j$ to $\varpi_{i+1}^j$. The last step of each phase is to establish the next nodes $\varpi_{i+2}^j$. If $\varpi_i^j$ sent a message to $\varpi_{i+1}^j$ then $\varpi_{i+1}^j$ checks its map and selects as $\varpi_{i+2}^j$ a node in $\varpi_{i+2}^j \in N^\tau(\varpi_{i+1}^j)$ that is farthest from $\varpi_i^j$. As an exception, if one of the corner representative nodes is in $N^\tau(\varpi_{i+1}^j) \setminus \{\varpi_i^j\}$, then this corner representative node is selected as $\varpi_{i+2}^j$ and then the process stops, i.e., $\varpi_{i+3}^j$ will not be selected.

Obviously, if there are no transmission conflicts between the eight pairs $\varpi_i^j$ and $\varpi_{i+1}^j$, then each phase can be performed in $2\tau$ communication steps (including sending the acknowledgements). The only way of having a transmission conflict is that two pairs $\varpi_i^j$ and $\varpi_{i+1}^j$, and $\varpi_i^{j'}$ and $\varpi_{i+1}^{j'}$, are transmitting along the same boundary and that in this phase $N^\tau(\varpi_i^j) \cap N^\tau(\varpi_i^{j'}) \neq \emptyset$. If this happen, then the nodes $\varpi_i^j$ and $\varpi_i^{j'}$ may not obtain an acknowledgement. In this case, both $\varpi_i^j$ and $\varpi_i^{j'}$ repeat the process of transmitting their messages to $\varpi_{i+1}^j$ and $\varpi_{i+1}^{j'}$, respectively, using the selector approach from Lemma 1 that ensures that the phase will be completed in $\mathcal{O}(\tau \cdot n^2 r^4 \log n) = \mathcal{O}(D)$ communication steps.

Let $\varrho_1$ and $\varrho_2$ be two adjacent corner representative nodes, and $(\varrho_2, \varpi_1^j, \varpi_2^j, \varpi_3^j, \ldots \varrho_1)$ be a sequence of nodes initialized by $\varrho_2$ in the process described before. It is easy to see that: *(i)* $\varrho_1$ receives all the messages of $\varrho_2, \varpi_1^j, \varpi_2^j, \varpi_3^j, \ldots$, *(ii)* $\varrho_2$ sends its message to all nodes in $\varpi_1^j, \varpi_2^j, \varpi_3^j, \ldots \varrho_1$, and *(iii)* for any boundary node $v$, there is a $\varpi_i^j$ such that $v \in N^\tau(\varpi_i^j)$ (which holds because of the way we pick $\varpi_i^j$).

Therefore, each corner representative node will receive all messages from the boundary nodes of its incident boundaries. If we repeat this process again, then each corner representative node will receive the messages of *all* boundary nodes. If we repeat this process once again, then all $\varpi_i^j$ nodes will receive the messages from all boundary nodes. If we now apply the approach from Lemma 1, then each boundary node will receive a message from at least one $\varpi_i^j$, and hence it will receive messages from all boundary nodes.

By our comments above, if there is no conflict in a phase, then the phase is completed in $2\tau$ communication steps, but if there is a conflict, then the number of communication steps in the phase is $\mathcal{O}(\tau\, n^2\, r^4 \log n)$. Now, we observe that if a corner representative node originates a transmission that should reach another corner representative node, then there will be at most a constant number of phases in which there will be a conflict. Therefore, the total running time for this algorithm is $\mathcal{O}(\tau \cdot D/\tau) + \mathcal{O}(\tau\, n^2\, r^4 \log n) = \mathcal{O}(D)$.

**Lemma 6.** *The algorithm above completes gossiping among all boundary nodes in $\mathcal{O}(D)$ time.*

*Gossiping via transmitting along almost parallel lines.* Let $\varrho$ be the corner representative node with the smallest ID. Let $\varrho^*$ be the corner representative node that shares the boundary with $\varrho$ (there are two such nodes) and that has the smaller ID. Let $\varrho$ select $\mathcal{O}(D/\tau)$ boundary nodes $\varsigma_1, \varsigma_2, \ldots$ such that *(i)* $\varsigma_{i+1} \in N^{\lfloor \tau/4 \rfloor}(\varsigma_i)$ for every $i$, and *(ii)* $\varsigma_j \notin N^{\lfloor \tau/32 \rfloor}(\varsigma_i)$ for every $i, j$, $i \neq j$. It is easy to see that such a sequence exists, and that $\varrho$ is able to determine it because after Lemma 6, $\varrho$ knows all boundary nodes and their $\tau$ neighbors. Next, $\varrho$ informs all boundary nodes about its choice using the process from the previous section. We now present an algorithm in which all the nodes $\varsigma_i$ will originate a procedure ***Straight-line transmission*** aiming at disseminating the information contained by these nodes along a line orthogonal to the boundary shared by $\varrho$ and $\varrho^*$.

There are a few problems with this approach that we need to address. First of all, we do not know the boundary of the unit square and instead, the goal will be to consider lines orthogonal to the line $\mathcal{L}$ going through $\varrho$ and $\varrho^*$. The location of $\mathcal{L}$ can be determined from the local map known to all the boundary nodes. Notice that since the angle between the boundary of $[0,1]^2$ and $\mathcal{L}$ is at most $\mathcal{O}(r)$, $\mathcal{L}$ is a good approximation of the boundary of $[0,1]^2$. Next, we observe that we will not be able to do any transmissions along any single line because our network $\mathcal{N}$ does not contain three collinear nodes with high probability. Therefore, our process will need to proceed along an approximate line. We begin with the following lemma that will help us quantify the angle between the perfect line we want to transmit along and the line along which we will actually transmit. The lemma easily follows from the Chernoff Bound.

**Lemma 7.** *Let* $\tau = \lceil 1/(n^2 r^5 \log n) \rceil$ *and* $r \leq \mathcal{O}\left(\frac{1}{n^{7/16} \log^{5/16} n}\right)$. *Let* $u$ *be a node in* $\mathcal{N}$ *and let* $\ell_u$ *be any ray (half-line) starting at* $u$. *If all points* $q \in \ell_u$ *with* $\mathsf{dist}(u,q) \leq \tau \cdot r$ *are contained in* $[0,1]^2$ *then with high probability there is a node* $w \in N^{\lfloor \tau/4 \rfloor}(u) \setminus N^{\lceil \tau/32 \rceil}(u)$ *such that* $|\measuredangle(\ell_u u w)| \leq O(\tau^2 r^2)$.

Now, we use Lemma 7 to design a scheme that allows a point to transmit a message along an approximate line. Our procedure Straight-line transmission$(s, \mu, \ell)$ aims at transmitting a message $\mu$ from node $s$ along (approximately) line $\ell_s$, $s \in \ell_s$, so that all nodes that are close to $\ell_s$ will receive the message $\mu$.

In Straight-line transmission$(s, \mu, \ell_s)$, the node $s$ initiates sending its message $\mu$ along the line $\ell_s$. The transmission process is performed in *phases*; each phase consists of sending a message from a node $\varpi_i$ to another node $\varpi_{i+1}$ such that $\ell_s$ is approximately equal to the line going through $\varpi_i$ and $\varpi_{i+1}$, and $\varpi_{i+1} \in N^{\lfloor \tau/4 \rfloor}(\varpi_i) \setminus N^{\lfloor \tau/32 \rfloor}(\varpi_i)$. The nodes $\varpi_i$ are determined recursively. Initially, $\varpi_0 = s$ and $\varpi_1$ is the node $q \in N^{\lfloor \tau/4 \rfloor}(s) \setminus N^{\lfloor \tau/32 \rfloor}(s)$ for which $|\measuredangle(\ell_s s q)|$ is minimized. If $i \geq 1$ and $\varpi_i$ is determined, then *(i)* if $N^{\tau}(\varpi_i) \setminus \{\varpi_{i-1}\}$ contains a boundary node then $\varpi_{i+1}$ is undefined and the process is stopped; *(ii)* otherwise, $\varpi_{i+1}$ is selected to be $u \in N^{\lfloor \tau/4 \rfloor}(\varpi_i) \setminus N^{\lfloor \tau/32 \rfloor}(\varpi_i)$ for which $|\measuredangle(\varpi_{i-1}\varpi_i\varpi_{i+1}) - \pi|$ is minimized. Since $\varpi_i$ knows the locations of all nodes in $N^{\tau}(\varpi_i)$, $\varpi_i$ is able to select $\varpi_{i+1}$ using its local map. Observe that by Lemma 7, for every node $\varpi_i$, $i \geq 1$, we have $|\measuredangle(\varpi_{i-1}\varpi_i\varpi_{i+1}) - \pi| \leq \mathcal{O}(\tau^2 r^2)$, with high probability. Next, since $\mathsf{dist}(\varpi_i \varpi_{i+1}) = \Theta(\tau \cdot r)$, we conclude that the last representative $\varpi_i$ will have index $\mathcal{O}(\frac{1}{\tau r})$. Hence, for every $i \geq 2$, we have $|\measuredangle(\ell_s u \varpi_i)| \leq \mathcal{O}(\frac{1}{\tau r} \cdot \tau^2 r^2) = \mathcal{O}(\tau r)$ with high probability. The running time of each phase of Straight-line transimmision is $O(\tau)$. So the running time of Straight-line transmission is $\mathcal{O}(\tau \cdot \frac{1}{\tau r}) = \mathcal{O}(D)$.

We will run multiple calls to Straight-line transmission$(s, \mu, \ell_s)$ with $s$ being the nodes $\varrho$, $\varrho^*$, and $\varsigma_1, \varsigma_2, \ldots$, as defined earlier and with line $\ell_s$ being the line going through $s$ that is orthogonal to the line $\mathcal{L}$ (which is the line going through $\varrho$ and $\varrho^*$).

It is easy to see in random geometric networks, if $u$ is the strict $k^{(th)}$ neighbor of $v$, then $dist(u,v) \geq k\, r/4$ with high probability. Hence the distance between any of the points $\varrho$, $\varrho^*$, and $\varsigma_1, \varsigma_2, \ldots$ is at least $\Omega(\tau\, r)$, so are the distance between the lines $\ell_s$. On the other hand, as we argued above, every procedure Straight-line transmission$(s, \mu, \ell_s)$ is sending messages only among the nodes that are at distance at most $O(\tau r)$ from the line $\ell_s$, where this claim holds with high probability. Therefore, in particular, the communication in the calls to Straight-line transmission$(s, \mu, \ell_s)$ will be done without any interference between the calls, with high probability (to avoid collisions between adjacent lines we interleave the transmissions in adjacent lines, yielding a $\mathcal{O}(1)$ factor slow-down).

**Lemma 8.** *All calls to Straight-line transmission$(s, \mu, \ell_s)$ with $s$ being $\varrho$, $\varrho^*$, and $\varsigma_1, \varsigma_2, \dots$ can be completed in $\mathcal{O}(D)$ communication steps, with high probability.*

Observe that while running the procedures Straight-line transmission, each node that is transmitting can include in its message also all the knowledge it contains at a given moment. Therefore, in particular, each last node $\varpi_k$ will receive all the messages collected on its path from $s$.

Next, let us observe that for every node $q$ in the network $\mathcal{N}$ either $q$ has been selected as one of the nodes $\varpi_i$ in one of the calls to Straight-line transmission or one of the nodes in $N^\tau(q)$ has. Indeed, since the distance between the adjacent lines $\ell_s$ is at most $\lfloor \tau/4 \rfloor \cdot r$, for each point $q \in \mathcal{N}$ there is a line $\ell_s$ with $\mathsf{dist}(q, \ell_s) \leq \lfloor \tau/4 \rfloor \cdot r/2$. Therefore, there will be at least one node $\varpi_i$ for Straight-line transmission$(s, \mu, \ell_s)$ with $\mathsf{dist}(q, \varpi_i) \leq \tau \cdot r/2$. This yields $\varpi_i \in N^\tau(q)$ with high probability. Because of this, if all nodes $u \in \mathcal{N}$ know the messages from all nodes in $N^\tau(u)$, then after completing the calls to Straight-line transmission, for each node $u \in \mathcal{N}$ there will be at least one boundary node that received the message of $u$.

If we do gossiping among the boundary nodes once again, all the boundary nodes will have the messages from all the nodes in $\mathcal{N}$. Next, we run again Straight-line transmission$(s, \mu, \ell_s)$ with $s$ being $\varrho$, $\varrho^*$, and $\varsigma_1, \varsigma_2, \dots$ as defined above. Then, all the nodes $\varpi_i$ will obtain the messages from all nodes in $\mathcal{N}$. Finally, since each $q \in \mathcal{N}$ has in its $\tau$-neighborhood a node $\varpi_i$, we can apply Lemma 1 to ensure that all nodes in $\mathcal{N}$ will receive the messages from all other nodes in $\mathcal{N}$.

**Theorem 2.** *Let $c\sqrt{\log n/n} \leq r \leq \mathcal{O}\left(\frac{1}{n^{7/16}\log^{5/16} n}\right)$. In the range-aware model, there is a deterministic distributed algorithm that completes gossiping in a random geometric network can be completed in deterministic time $\mathcal{O}(D)$. The algorithm may fail with probability at most $1/n^2$.*

## 5 Deterministic Distributed Algorithm: Knowing Angles Helps

One can modify the algorithm from Theorem 2 to work in the scenario in which a node cannot determine the distance between its neighboring node but instead, it is able to determine the relative direction where the neighbor is located.

**Theorem 3.** *Let* $c\sqrt{\log n/n} \le r \le \mathcal{O}\left(\frac{1}{n^{7/16}\log^{5/16} n}\right)$. *If each node receiving the message is able to determine the relative direction from which the message arrives, then gossiping in a random geometric network can be completed in deterministic time* $\mathcal{O}(D)$. *The algorithm may fail with probability at most* $1/n^2$.

The algorithm is essentially the same as that described in Section 4 with two differences. First of all, now the local map of a node does not have the exact distances but it may be re-scaled. That is, using the same approach as presented in Section 4, each node can build its local map where all the angles in the map are the actual angles between the points, but only the distances may be re-scaled. Secondly, we need another approach to determine if a node is a boundary node or it is a corner node. This can be done by comparing the density of the neighborhoods of the nodes (Details deferred to the full version.)

## 6 Deterministic Distributed Algorithm: Knowing Locations Helps

We consider also the gossiping problem in random geometric networks in the power model, where each node knows its geometric position in the unit square. In such model, Dessmark and Pelc [9] give a deterministic algorithm for *broadcasting* that (in our setting) runs in $\mathcal{O}(D)$ time. We can prove a similar result for *gossiping* by extending the preprocessing phase from [9] and use an appropriate strongly-selective family to collect information about the neighbors of each point (Details deferred to the full version.)

**Theorem 4.** *If every input node knows its location* $[0,1]^2$, *then there is a deterministic algorithm that completes gossiping in a random geometric network in time* $\mathcal{O}(n^2\, r^4\, \log n + 1/r)$. *In particular, if* $r \le \mathcal{O}\left(\frac{1}{n^{2/5}\log^{1/5} n}\right)$ *then the running time is* $\mathcal{O}(D)$. *The algorithm may fail with probability at most* $1/n^2$.

## 7 Conclusions

In this paper we presented the first thorough study of basic communication aspects in random geometric ad-hoc radio networks. We have shown that in many scenarios, the random structure of these networks (which often may model well realistic scenarios from sensor networks) allows us to perform communication between the nodes in the network in asymptotically optimal time $\mathcal{O}(D)$, where $D$ is the diameter of the network and thus a trivial lower bound for any communication. This is in contrast to arbitrary ad-hoc radio networks, where deterministic bounds of $o(n)$ are unattainable.

Our study shows also that while there is a relatively simple optimal randomized gossiping algorithm and a deterministic one when the nodes have knowledge about their locations in the plane, the other scenarios are more complicated. In particular, we do not know if $\mathcal{O}(D)$-time deterministic gossiping is possible in the unknown topology model.

## References

1. Bar-Yehuda, R., Goldreich, O., Itai, A.: On the time-complexity of broadcast in multi-hop radio networks: An exponential gap between determinism and randomization. Journal of Computer and System Sciences 45(1), 104–126 (1992)
2. Capkun, S., Hamdi, M., Hubaux, J.-P.: GPS-free positioning in mobile ad hoc networks. Cluster Computing 5(2), 157–167 (2002)
3. Clementi, A.E.F., Monti, A., Silvestri, R.: Distributed broadcast in radio networks of unknown topology. Theoretical Computer Science 302, 337–364 (2003)
4. Chlebus, B.S., Gąsieniec, L., Gibbons, A., Pelc, A., Rytter, W.: Deterministic broadcasting in ad hoc radio networks. Distributed Computing 15(1), 27–38 (2002)
5. Chlebus, B.S., Kowalski, D.R., Rokicki, M.A.: Average-time coplexity of gossiping in radio networks. In: Flocchini, P., Gkasieniec, L. (eds.) SIROCCO 2006. LNCS, vol. 4056, pp. 253–267. Springer, Heidelberg (2006)
6. Chrobak, M., Gąsieniec, L., Rytter, W.: Fast broadcasting and gossiping in radio networks. Journal of Algorithms 43(2), 177–189 (2002)
7. Czumaj, A., Rytter, W.: Broadcasting algorithms in radio networks with unknown topology. Journal of Algorithms 60(2), 115–143 (2006)
8. Czumaj, A., Wang, X.: Communication problems in random line-of-sight ad-hoc radio networks. In: Proc 4th Symposium on Stochastic Algorithms, Foundations, and Applications (SAGA), pp. 70–81 (2007)
9. Dessmark, A., Pelc, A.: Broadcasting in geometric radio networks. Journal of Discrete Algorithms 5(1), 187–201 (2007)
10. Elsässer, R., Gąsieniec, L.: Radio communication in random graphs. Journal of Computer and System Sciences 72, 490–506 (2006)
11. Gąsieniec, L., Peleg, D., Xin, Q.: Faster communication in known topology radio networks. In: Proc. 24th PODC, pp. 129–137 (2005)
12. Gąsieniec, L., Radzik, T., Xin, Q.: Faster deterministic gossiping in directed adhoc radio networks. In: Hagerup, T., Katajainen, J. (eds.) SWAT 2004. LNCS, vol. 3111, pp. 397–407. Springer, Heidelberg (2004)
13. Giordano, S., Stojmenovic, I.: Position based ad hoc routes in ad hoc networks. In: Handbook of Ad Hoc Wireless Networks. ch. 16, pp. 1–14 (2003)
14. Gupta, P., Kumar, P.R.: Critical power for asymptotic connectivity in wireless networks. In: Stochastic Analysis, Control, Optimization and Applications: A Volume in Honor of W. H. Fleming, pp. 547–566 (1998)
15. Gupta, P., Kumar, P.R.: The capacity of wireless networks. IEEE Transactions on Information Theory IT-46, 388–404 (2000)
16. Ko, Y.B., Vaidya, N.H.: Location aided routing (LAR) in mobile adhoc networks. In: Proc 4th Annual ACM/IEEE International Conference on Mobile Computing and Networking (MOBICOM), pp. 66–75 (1998)
17. Kowalski, D., Pelc, A.: Broadcasting in undirected ad hoc radio networks. Distributed Computing 18(1), 43–57 (2005)
18. Kowalski, D., Pelc, A.: Optimal deterministic broadcasting in known topology radio networks. Distributed Computing 19(3), 185–195 (2007)
19. Li, X., Shi, H., Shang, Y.: A partial-range-aware localization algorithm for Ad-hoc wireless sensor networks. In: Proc. 29th Annual IEEE International Conference on Local Computer Networks (2004)
20. Nasipuri, A., Li, K., Sappidi, U.R.: Power consumption and throughput in mobile ad hoc networks using directional antennas. In: Proc. 11th IEEE International Conf. on Computer Communications and Networks (ICCCN), pp. 620–626 (2002)
21. Penrose, M.D.: Random Geometric Graphs. Oxford University Press, UK (2003)