Prof. Dr. Christian Schindelhauer Freiburg i. Br., 09/09/2009

Exam Algorithms Theory

Winter 2008/09

Name : ....

Matriculation number : ....

Program of study (Studiengang) : ....

Result (do not write here!)

Task 1		of 15	
Task 2		of 15	
Task 3		of 15	
Task 4		of 15	
Task 5		of 15	
Task 6		of 15	
Task 7		of 15	
Bonus points		of 15	
Sum		of 90	Top 6

Grade:

The exam consists of 7 tasks and 24 pages. From these 7 tasks and the bonus points of the exercise class the 6 highest numbers will account for the result. The maximum reachable number of points is 90; the exam is passed with at least 45 points.

Please write your matriculation number on each sheet.

Please write your solution in the boxes below the tasks. If the space is not sufficient, you can ask for additional sheets.

Non-programmable calculators and an A4 sheet with hand-written notes (double-sided) are permitted.

Task 1: Divide and Conquer **15 points**

Consider the following problem: Given n points $(x_1, y_1), \ldots, (x_n, y_n)$ in the two-dimensional Euclidean plane, find the closest pair of points, i.e. i, j minimizing

$$\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}\ .$$

1. Present an efficient algorithm with running time $o(n^2)$ (less than quadratic).

2. Give the run-time analysis of your algorithm.

Matriculation number:

Task 2: FFT

15 points

Compute the product of the two polynomials $p(x) = 2x - 1$ and $q(x) = 2x + 1$ using the Fast Fourier Transformation.

1. Compute the FFT of $p(x)$ and $q(x)$ using an appropriate choice of k (for the k-th roots of unity).
2. Give the point-value representation of pq at the k-th roots of unity.
3. Compute the interpolation by using the FFT algorithm.
4. Check the correctness of your result.

Specify all (recursive) calls of FFT algorithm as well as the outputs used during the execution.

Matriculation number:

Task 3: RSA **15 points**

For an RSA encryption choose $p = 7$ and $q = 11$. Let $e = 7$.

1. Compute the number $d = e^{-1} \bmod pq$ using the algorithm *Extended-Euclid*.

2. Present the public and private key of RSA.

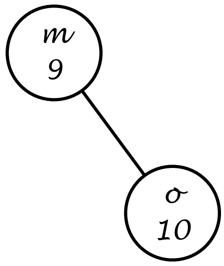
3. Encode the message $m = 17$ using the *Fast Exponentiation Algorithm*.

Task 4: Treaps **15 points**

1. Sequentially insert the elements (i, 8), (j, 4), (k, 11), (h, 2) and (g, 5) into an initially empty Treap. For all intermediate stages, e.g after performing a rotation, illustrate the state of the Treap and specify the operation that leads to this state.

2. Delete the root of the Treap of the tree. Illustrate the Treap prior to and after each rotation.

3. Merge the above Treap and the Treap shown below. Illustrate all intermediate stages.



Task 5: Bin Packing

15 points

Consider the following input sequence:

$$\underbrace{\frac{1}{6}-2\epsilon,\ldots,\frac{1}{6}-2\epsilon}_{6m},\underbrace{\frac{1}{3}+\epsilon,\ldots,\frac{1}{3}+\epsilon}_{6m},\underbrace{\frac{1}{2}+\epsilon,\ldots,\frac{1}{2}+\epsilon}_{6m}$$

1. Which is the optimal bin packing solution?

2. Which solution is computed by *Next Fit*?

3. Which solution is computed by *First Fit*?

4. Which solution is computed by *Best Fit*?

5. Which solution is computed by *First Fit Decreasing*?

Matriculation number:

Task 6: Binomial Queue **15 points**

1. Insert 56, 34, 2, 7, 1, 27, 5, 16 in an empty binominal queue. Show all intermediate results.

Matriculation number:

2. Perform a *deletemin* operation.

Task 7: Dynamic Programming **15 points**

Compute the optimal Parenthesization of a Matrix Chain Multiplication consisting of four matrices with dimensions specified by the sequence (6, 10, 5, 2, 20), i.e. with matrices $A_1(6 \times 10), A_2(10 \times 5), A_3(5 \times 2), A_4(2 \times 20)$.

Use the method from the lecture and outline the intermediate results. Present the optimal parenthesization and its run-time.

Matriculation number: