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# Algorithm Theory

## 3 Fast Fourier Transformation


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Algorithms Theory

Chapter 3

# Fast Fourier Transformation

# Polynomials

**Polynomials *p* over real numbers with a variable *x***

$$p(x) = a_n x^n + \ldots + a_1 x^1 + a_0$$

$a_0, \ldots, a_n \in R$, $a_n \neq 0$: **coefficients of** $p$

**n = degree of** $p$**: highest power of x in** $p$

**Example:**

$p(x) = 3x^3 - 15x^2 + 18x$

**Set of all polynomials over real polynomials:** $R[x]$

# 2. Operations on Polynomials

$p,q \in R[x]$

$$p(x) = a_n x^n + \ldots + a_1 x^1 + a_0$$
$$q(x) = b_n x^n + \ldots + b_1 x^1 + b_0$$

## 1. Addition

$$\begin{aligned} p(x) + q(x) &= (a_n x^n + \ldots + a_0) + (b_n x^n + \ldots + b_0) \\ &= (a_n + b_n) x^n + \ldots + (a_1 + b_1) x^1 + (a_0 + b_0) \end{aligned}$$

# Operation on Polynomials

**2. Multiplication**

$$p(x)q(x) = (a_n x^n + \ldots + a_0)(b_n x^n + \ldots + b_0)$$
$$= c_{2n} x^{2n} + \ldots + c_1 x^1 + c_0$$

$c_i$**: Which monomial products have degree** ***i*** **?**

$$\Rightarrow c_i = \sum_{j=0}^{i} a_j b_{i-j} \quad i = 0,\ldots,2n.$$

$$a_{n+1} = \ldots = a_{2n} = 0, b_{n+1} = \ldots = b_{2n} = 0$$

**Polynomial ring** *R[x]*

# Operation on Polynomials

**3. Evaluation at $x_0$: Horner-Schema**

$$p(x_0) = (\ldots(a_n x_0 + a_{n-1})x_0 + \ldots + a_1)x_0 + a_0$$

**Runtime: O(n)**

# Representations of Polynomials

$p(x) \in R[x]$

**Possible representations of $p(x)$:**

**1. Coefficient representation**

$$p(x) = a_n x^n + \ldots + a_1 x^1 + a_0$$

**Example:**

$$p(x) = 3x^3 - 15x^2 + 18x$$

# Representations of Polynomials

**2. Root representation**

*p(x) ∈ R[x]*

$$p(x) = a_n (x - x_1)\ldots(x - x_n)$$

**Beispiel:**

$$p(x) = 3x(x-2)(x-3)$$

# Representations of Polynomials

**3. Point-value representation**

**Interpolation lemma**

Any polynomial p(x) over *R[x]* of degree *n* is uniquely defined by n+1 pairs **$(x_i, p(x_i))$, where $i = 0,\ldots,n$ and $x_i \neq x_j$ für $i \neq j$**

**Beispiel:**

The polynomial

$$p(x) = 3x(x-2)(x-3)$$

is determined by the point-values (0,0), (1,6), (2,0), (3,0).

# Operations on Polynomials

*p, q* $\in$ *R[x]*, **Grad(*p*) = Grad(*q*) = *n***

- **Coeffient representation**

  **Addition: O(*n*)**

  **Product: O(*n*$^2$)**

  **Evaluation at $x_0$: O(*n*)**

- **Point-value representation**

$$p = (x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$$

$$q = (x_0, z_0), (x_1, z_1), \ldots, (x_n, z_n)$$

# Operations on Polynomials

**Addition:**

$$p+q=(x_0,y_0+z_0),(x_1,y_1+z_1),\ldots,(x_n,y_n+z_n)$$

Runtime: O(*n*)

**Multiplication:**

$$p\cdot q=(x_0,y_0\cdot z_0),(x_1,y_1\cdot z_1),\ldots,(x_n,y_n\cdot z_n)$$

(Condition: $n \geq$ degree($pq$))

Runtime: O(n)

**Evaluation at** ***x´:***

Convert polynomial to coefficient representation

(Interpolation)

# Polynomial Multiplication

**Multiplication of two polynomials *p, q* of degree < *n***

$p,q$ of degree *n-1*, *n* coefficients

$\downarrow$ **Evaluation:** $x_0, x_1, \ldots, x_{2n-1}$

$2n$ point-value pairs $(x_i, p(x_i))$ and $(x_i, q(x_i))$

$\downarrow$ **Pointwise multiplication**

$2n$ point-value pairs $(x_i, pq(x_i))$

$\downarrow$ **Interpolation**

$pq$ of degree $2n$-2, $2n$-1 coefficients

# Divide and Conquer Approach

**Idea:** (for even *n*)

$$
\begin{aligned}
p(x) &= a_0 + a_1 x + \ldots + a_{n-1} x^{n-1} \\
&= a_0 + a_2 x^2 + \ldots + a_{n-2} x^{n-2} + \\
&\quad a_1 x + a_3 x^3 + \ldots + a_{n-1} x^{n-1} \\
&= a_0 + a_2 x^2 + \ldots + a_{n-2} \left(x^2\right)^{(n-2)/2} + \\
&\quad x\left(a_1 + a_3 x^2 + \ldots + a_{n-1} \left(x^2\right)^{(n-2)/2}\right) \\
&= p_0\left(x^2\right) + x p_1\left(x^2\right)
\end{aligned}
$$

$$p_0(x) = a_0 + a_2 x + \ldots + a_{n-2} x^{(n-2)/2}$$
$$p_1(x) = a_1 + a_3 x + \ldots + a_{n-1} x^{(n-2)/2}$$

Select $x_0, \ldots, x_{2n-1}$ such that the computations of $p(x_k)$ and $p(x_{k+n})$ are almost identical.

# Representations of *p(x)*

**Assume:** degree(*p*) < *n*

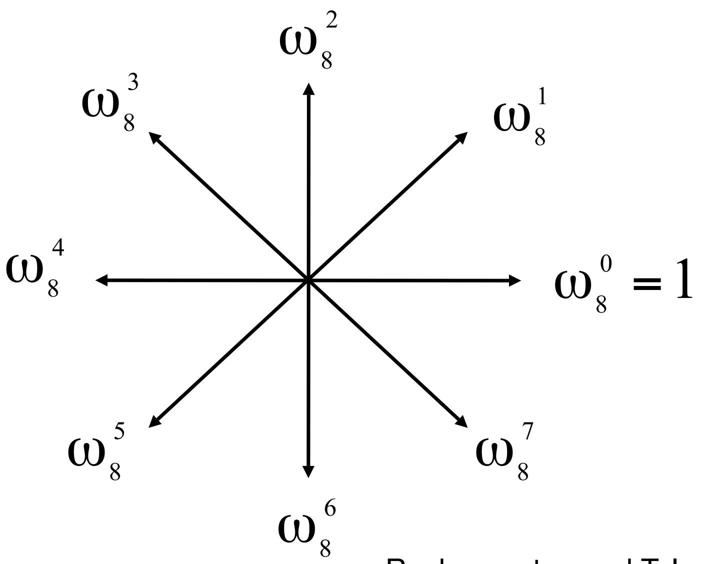
3a. **Values of the *n* powers of the principial *n*th root of unity**

$$\omega_n = e^{2\pi i/n}$$

$i = \sqrt{-1} \qquad e^{2\pi i} = 1$

Power of $\omega_n$ (roots of unity):

$$1 = \quad \omega_n^0, \omega_n^1, \ldots, \omega_n^{n-1}$$



# Discrete Fourier Transform

**The values $p(\omega_n^i)$ uniquely define p if degree($p$)<$n$.**

**Discrete Fourier Transformation (DFT)**

$$DFT_n(p) = \left(p(\omega_n^0), p(\omega_n^1), \ldots, p(\omega_n^{n-1})\right)$$

**Example:** $n$**=4**

$$e^{ix} = \cos x + i \sin x$$

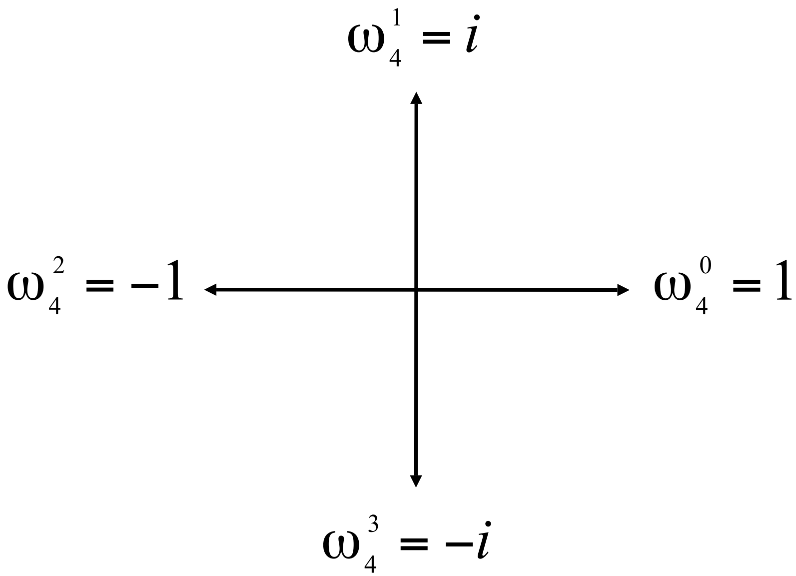
$$\omega_4^0 = e^{0i} = \cos(0) + i\sin(0) = 1$$

$$\omega_4^1 = e^{2\pi i/4} = \cos(\pi/2) + i\sin(\pi/2) = i$$

$$\omega_4^2 = \left(e^{2\pi i/4}\right)^2 = \cos\pi + i\sin\pi = -1$$

$$\omega_4^3 = \left(e^{2\pi i/4}\right)^3 = \cos(3\pi/2) + i\sin(3\pi/2) = -i$$

# Evaluation at the Unity Roots



# Evaluation at the Unity Roots

$$p(x) = 3x^3 - 15x^2 + 18x$$

$$\left(\omega_4^0, p(\omega_4^0)\right) = \left(1, p(1)\right) = \left(1,6\right)$$

$$\left(\omega_4^1, p(\omega_4^1)\right) = \left(i, p(i)\right) = \left(i, 15 + 15i\right)$$

$$\left(\omega_4^2, p(\omega_4^2)\right) = \left(-1, p(-1)\right) = \left(-1, -36\right)$$

$$\left(\omega_4^3, p(\omega_4^3)\right) = \left(-i, p(-i)\right) = \left(-i, 15 - 15i\right)$$

$$DFT_4(p) = \left(6, 15 + 15i, -36, 15 - 15i\right)$$

# Multiplication of Polynomials

**Computation of the product of *p* and *q* of degree < *n***

$p,q$ **of degree *n-1*, *n* coefficients**

↓ **Evaluation:** $\omega_{2n}^{0}, \omega_{2n}^{1}, \ldots, \omega_{2n}^{2n-1}$

$2n$ point-value pairs $\left(\omega_{2n}^{i}, p\left(\omega_{2n}^{i}\right)\right)$ and $\left(\omega_{2n}^{i}, q\left(\omega_{2n}^{i}\right)\right)$

↓ **Pointwise multiplication**

$2n$ point-value pairs $\left(\omega_{2n}^{i}, pq\left(\omega_{2n}^{i}\right)\right)$

↓ **Interpolation**

$pq$ of degree $2n$-2, $2n$-1 coefficients

# Properties of the Unity Roots

$\omega_{2n}^{0}, \omega_{2n}^{1}, \ldots, \omega_{2n}^{2n-1}$ form a multiplicative group

**Cancellation lemma**
For all $n > 0$, $0 \leq k \leq n$, and $d > 0$ we have

$$\omega_{dn}^{dk} = \omega_{n}^{k}$$

**Proof:**

$$\omega_{dn}^{dk} = e^{2\pi i dk/(dn)} = e^{2\pi i k/n} = \omega_{n}^{k}$$

Therefore:

$$\omega_{2n}^{n} = \omega_{2}^{1} = -1$$

# Discrete Fourier Transform

$$DFT_n(p) = \left(p(\omega_n^0), p(\omega_n^1), \ldots, p(\omega_n^{n-1})\right)$$

**Fast Fourier Transform:**

Computation of $DFT_n(p)$ using Divide-and-Conquer

# Discrete Fourier Transform

**Idee:** (for even *n*)

$$
\begin{aligned}
p(x) &= a_0 + a_1 x + \ldots + a_{n-1} x^{n-1} \\
&= a_0 + a_2 x^2 + \ldots + a_{n-2} x^{n-2} + \\
&\quad a_1 x + a_3 x^3 + \ldots + a_{n-1} x^{n-1} \\
&= a_0 + a_2 x^2 + \ldots + a_{n-2} \left(x^2\right)^{(n-2)/2} + \\
&\quad x\left(a_1 + a_3 x^2 + \ldots + a_{n-1} \left(x^2\right)^{(n-2)/2}\right) \\
&= p_0\left(x^2\right) + x p_1\left(x^2\right)
\end{aligned}
$$

$$
\begin{aligned}
p_0(x) &= a_0 + a_2 x + \ldots + a_{n-2} x^{(n-2)/2} \\
p_1(x) &= a_1 + a_3 x + \ldots + a_{n-1} x^{(n-2)/2}
\end{aligned}
$$

# Discrete Fourier Transform

**Evaluation for *k* = 0, ... , *n* – 1**

$$p(\omega_n^k) = p_0\left((\omega_n^k)^2\right) + \omega_n^k p_1\left((\omega_n^k)^2\right) = \begin{cases} p_0\left(\omega_{n/2}^k\right) + \omega_n^k p_1\left(\omega_{n/2}^k\right), \\ \text{if } k < n/2 \\ p_0\left(\omega_{n/2}^{k-n/2}\right) + \omega_n^k p_1\left(\omega_{n/2}^{k-n/2}\right), \\ \text{if } k \geq n/2 \end{cases}$$

$$\begin{aligned} DFT_n(p) &= (p_0(\omega_{n/2}^0), \ldots, p_0(\omega_{n/2}^{n/2-1}),\ \ p_0(\omega_{n/2}^0), \ldots, p_0(\omega_{n/2}^{n/2-1})) \\ &+ (\omega_n^0 p_1(\omega_{n/2}^0), \ldots, \omega_n^{n/2-1} p_1(\omega_{n/2}^{n/2-1}),\ \ \omega_n^{n/2} p_1(\omega_{n/2}^0), \ldots, \omega_n^{n-1} p_1(\omega_{n/2}^{n/2-1})) \end{aligned}$$

# Discrete Fourier Transform

**Example:**

$$p(\omega_4^0) = p_0(\omega_2^0) + \omega_4^0 p_1(\omega_2^0)$$

$$p(\omega_4^1) = p_0(\omega_2^1) + \omega_4^1 p_1(\omega_2^1)$$

$$p(\omega_4^2) = p_0(\omega_2^0) + \omega_4^2 p_1(\omega_2^0)$$

$$p(\omega_4^3) = p_0(\omega_2^1) + \omega_4^3 p_1(\omega_2^1)$$

# Computation of $DFT_n$

$$DFT_n(p) = \left(p(\omega_n^0), p(\omega_n^1), \ldots, p(\omega_n^{n-1})\right)$$

**Simple case: $n$ = 1 (degree($p$) = $n$ –1 = 0)**

$DFT_1(p) = a_0$

**General case :**

**Divide:**

Divide $p$ in $p_0$ and $p_1$

**Conquer:**

Compute $DFT_{n/2}(p_0)$ and $DFT_{n/2}(p_1)$ recursively

**Merge:**

Compute for $k$ = 0, ... , $n$ –1:

$$DFT_n(p)_k = (DFT_{n/2}(p_0), DFT_{n/2}(p_0))_k + \omega_n^k \times (DFT_{n/2}(p_1), DFT_{n/2}(p_1))_k$$

# Further Improvement

$$p(\omega_n^k) = \begin{cases} p_0(\omega_{n/2}^k) + \omega_n^k p_1(\omega_{n/2}^k) & \text{if } k < n/2 \\ p_0(\omega_{n/2}^{k-n/2}) + \omega_n^k p_1(\omega_{n/2}^{k-n/2}) & \text{if } k \geq n/2 \end{cases}$$

$$= \begin{cases} p_0(\omega_{n/2}^k) + \omega_n^k p_1(\omega_{n/2}^k) & \text{if } k < n/2 \\ p_0(\omega_{n/2}^{k-n/2}) - \omega_n^{k-n/2} p_1(\omega_{n/2}^{k-n/2}) & \text{if } k \geq n/2 \end{cases}$$

Hence, if $k < n/2$:

$$p_0(\omega_{n/2}^k) + \omega_n^k p_1(\omega_{n/2}^k) = p(\omega_n^k)$$

$$p_0(\omega_{n/2}^k) - \omega_n^k p_1(\omega_{n/2}^k) = p(\omega_n^{k+n/2})$$

# Further Improvement

**Example:**

$$p(\omega_4^0) = p_0(\omega_2^0) + \omega_4^0 p_1(\omega_2^0)$$

$$p(\omega_4^1) = p_0(\omega_2^1) + \omega_4^1 p_1(\omega_2^1)$$

$$p(\omega_4^2) = p_0(\omega_2^0) - \omega_4^0 p_1(\omega_2^0)$$

$$p(\omega_4^3) = p_0(\omega_2^1) - \omega_4^1 p_1(\omega_2^1)$$

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# Fast Fourier Transform

**Algorithm** *FFT*

**Input:** **An Array** *a* **with** *n* **coefficients of a polynomial** *p* **and** $n = 2^k$

**Output:** $DFT_n(p)$

1. **if** $n = 1$ **then** /* *p ist constant* */
2. **return** *a*
3. $d^{[0]} = FFT([a_0, a_2, \ldots, a_{n-2}], n/2)$
4. $d^{[1]} = FFT([a_1, a_3, \ldots, a_{n-1}], n/2)$
5. $\omega_n = e^{2\pi i/n}$
6. $\omega = 1$
7. **for** $k = 0$ **to** $n/2 - 1$ **do** /* $\omega = \omega_n^k$ */
8. $d_k = d_k^{[0]} + \omega \times d_k^{[1]}$
9. $d_{k+n/2} = d_k^{[0]} - \omega \times d_k^{[1]}$
10. $\omega = \omega_n \times \omega$
11. **return** *d*

# FFT : Example

$$p(x) = 3x^3 - 15x^2 + 18x + 0$$

$a$ = [0, 18, -15, 3 ]

$a^{[0]}$ = [0, -15] $a^{[1]}$ = [18, 3]

*FFT*([0, -15], 2) = (FFT([0],1) + FFT([-15],1), FFT([0],1) - FFT([-15],1))

= (-15,15)

*FFT*([18, 3],2) = (FFT([18],1) + FFT([3],1), FFT([18],1) - FFT([3],1))

= (21,15)

$k = 0$ ; $\omega = 1$

$d_0 = -15 + 1 * 21 = 6$ $d_2 = -15 - 1 * 21 = -36$

$k = 1$ ; $\omega = i$

$d_1 = 15 + i*15$ $d_3 = 15 - i*15$

*FFT*($a$, 4) = (6, 15+15i, -36, 15 –15i)

# Analysis

**T(n) = running time for evaluating a polynomial of degree< n at n positions** $\omega_{2n}^{0}, \omega_{2n}^{1}, \ldots, \omega_{2n}^{2n-1}$

**T(1) = O(1)**

**T(n) = 2 T(n/2) + O(n)**

**= O(n log n)**

# Polynomial Multiplication

Compute the product of two polynomials *p, q* of degree < *n*:

$p,q$ of degree *n-1*, *n* coefficients

$\downarrow$ **Evaluation by FFT:** $\omega_{2n}^0, \omega_{2n}^1, \ldots, \omega_{2n}^{2n-1}$

$2n$ point-value pairs $(\omega_{2n}^i, p(\omega_{2n}^i))$ und $(\omega_{2n}^i, q(\omega_{2n}^i))$

$\downarrow$ **Pointwise multiplication**

$2n$ point-value pairs $(\omega_{2n}^i, pq(\omega_{2n}^i))$

$\downarrow$ **Interpolation via FFT**

$pq$ of degree $2n$-2, $2n$-1 coefficients

# Interpolation

**Convert the point-value representation into coefficient representation.**

**Input: $(x_0, y_0),\ldots, (x_{n-1}, y_{n-1})$ mit $x_i \neq x_j$, für alle $i \neq j$**

**Output:** Polynomial *p* with coefficients $a_0,\ldots, a_{n-1}$, such that

$$\begin{aligned}
p(x_0) &= a_0 + a_1 x_0 + \ldots + a_{n-1} x_0^{n-1} &= y_0 \\
p(x_1) &= a_0 + a_1 x_1 + \ldots + a_{n-1} x_1^{n-1} &= y_1 \\
p(x_2) &= a_0 + a_1 x_2 + \ldots + a_{n-1} x_2^{n-1} &= y_2 \\
&\vdots & \vdots \\
p(x_{n-1}) &= a_0 + a_1 x_{n-1} + \ldots + a_{n-1} x_{n-1}^{n-1} &= y_{n-1}
\end{aligned}$$

# Interpolation

**Matrix notation:**

$$\begin{pmatrix} 1 & x_0 & \cdots & x_0^{n-1} \\ 1 & x_1 & \cdots & x_1^{n-1} \\ & & \vdots & \\ 1 & x_{n-1} & \cdots & x_{n-1}^{n-1} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_{n-1} \end{pmatrix}$$

# Interpolation

System of equations

$$\begin{pmatrix} 1 & x_0 & \cdots & x_0^{n-1} \\ 1 & x_1 & \cdots & x_1^{n-1} \\ & & \vdots & \\ 1 & x_{n-1} & \cdots & x_{n-1}^{n-1} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_{n-1} \end{pmatrix}$$

solvable if $x_i \neq x_j$ for all $i \neq j$.

**Special Case** (here) : $x_i = \omega_n^i$

**Definition:** $V_n = \left(\omega_n^{ij}\right)_{i,j}, \; a = (a_i), \quad y = (y_i)$

$$V_n a = y \quad \Rightarrow \quad a = V_n^{-1} y$$

# Interpolation

**Theorem**
For all 0 ≤ *i, j* ≤ *n* – 1 we have

$$\left(V_n^{-1}\right)_{ij} = \frac{\omega_n^{-ij}}{n}$$

**Proof**

$$V_n^{-1} = \left(\frac{\omega_n^{-ij}}{n}\right)_{i,j}$$

We have to show:

$$V_n^{-1} V_n = I_n$$

# Interpolation

Consider the entry of $V_n^{-1}V_n$ in line $i$ and column $j$ :

$$\left(V_n^{-1}V_n\right)_{ij} =$$

$$\begin{pmatrix} & & \cdots & \\ \frac{1}{n} & \frac{\omega_n^{-i}}{n} & \cdots & \frac{\omega_n^{-i(n-1)}}{n} \\ & & \vdots & \\ & & \cdots & \end{pmatrix} \begin{pmatrix} \cdots & 1 & \cdots \\ \cdots & \omega_n^{j} & \cdots \\ \cdots & \omega_n^{2j} & \cdots \\ & \vdots & \\ \cdots & \omega_n^{(n-1)j} & \cdots \end{pmatrix}_{ij}$$

# Interpolation

$$\left(V_n^{-1}V_n\right)_{ij} = \sum_{k=0}^{n-1}\frac{\omega_n^{-ik}}{n}\omega_n^{jk} = \frac{1}{n}\sum_{k=0}^{n-1}\omega_n^{(-i+j)k}$$

**Case 1:** $i = j$

$$\frac{1}{n}\sum_{k=0}^{n-1}\omega_n^{(-i+j)k} = \frac{1}{n}\sum_{k=0}^{n-1}\omega_n^{0\cdot k} = 1$$

**Case 2:** $i \neq j$, i.e. $-(n-1) \leq -i + j \leq n-1$

thus $n \nmid -i+j$ :

$$\frac{1}{n}\sum_{k=0}^{n-1}\omega_n^{(-i+j)k} = 0$$

# Interpolation

**Summation lemma:**

For any integer $n > 0$, $l \geq 0$ with $n \nmid l$:

$$\sum_{k=0}^{n-1} \omega_n^{lk} = 0$$

**Proof:**

$$\sum_{k=0}^{n-1} \left(\omega_n^l\right)^k = \frac{\left(\omega_n^l\right)^n - 1}{\omega_n^l - 1} = \frac{\left(\omega_n^n\right)^l - 1}{\omega_n^l - 1} = 0$$

# Interpolation

$$a_i = \left(V_n^{-1} y\right)_i$$

$$= \left(\frac{1}{n}, \frac{\omega_n^{-i}}{n}, \ldots, \frac{\omega_n^{-i(n-1)}}{n}\right) \begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_{n-1} \end{pmatrix}$$

$$= \sum_{k=0}^{n-1} y_k \frac{\omega_n^{-ik}}{n}$$

$$= \frac{1}{n} \sum_{k=0}^{n-1} y_k \left(\omega_n^{-i}\right)^k$$

# Interpolation

$$a = \frac{1}{n}\left(\sum_{k=0}^{n-1} y_k \left(\omega_n^{-0}\right)^k, \sum_{k=0}^{n-1} y_k \left(\omega_n^{-1}\right)^k, \cdots, \sum_{k=0}^{n-1} y_k \left(\omega_n^{-(n-1)}\right)^k\right)$$

$$r(x) = y_0 + y_1 x + y_2 x^2 + \cdots + y_{n-1} x^{n-1}$$

$$a = \frac{1}{n}\left(r\left(\omega_n^{-0}\right), r\left(\omega_n^{-1}\right), \ldots, r\left(\omega_n^{-(n-1)}\right)\right)$$

# Interpolation and DFT

$$a = \frac{1}{n}\left(r\left(\omega_n^{-0}\right), r\left(\omega_n^{-1}\right), \ldots, r\left(\omega_n^{-(n-1)}\right)\right)$$

$$a = \frac{1}{n}\left(r\left(\omega_n^{n}\right), r\left(\omega_n^{n-1}\right), \ldots, r\left(\omega_n^{1}\right)\right) \qquad \text{since } \omega_n^n = 1$$

$$a_i = \frac{1}{n}\left(DFT_n(r)\right)_{n-i} \qquad (i \neq 0)$$

$$a_0 = \frac{1}{n}\left(DFT_n(r)\right)_0$$

# Polynomial Multiplication

Compute the product of two polynomials *p, q* of degree < *n*:

$p,q$ of degree *n-1*, *n* coefficients

$\downarrow$ **Evaluation by FFT:** $\omega_{2n}^{0}, \omega_{2n}^{1}, \ldots, \omega_{2n}^{2n-1}$

2$n$ point-value pairs $\left(\omega_{2n}^{i}, p\left(\omega_{2n}^{i}\right)\right)$ und $\left(\omega_{2n}^{i}, q\left(\omega_{2n}^{i}\right)\right)$

$\downarrow$ **Pointwise multiplication**

2$n$ point-value pairs $\left(\omega_{2n}^{i}, pq\left(\omega_{2n}^{i}\right)\right)$

$\downarrow$ **Interpolation via FFT**

$pq$ of degree 2$n$-2, 2$n$-1 coefficients

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# Algorithm Theory

## 3 Fast Fourier Transform

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