ALBERT-LUDWIGS-UNIVERSITÄT FREIBURG

Algorithm Theory

5 Randomized Algorithms: Public Key Cryptosystems

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Randomized algorithms

- Classes of randomized algorithms
- Randomized Quicksort
- Randomized primality test
- Cryptography

Classes of randomized algorithms

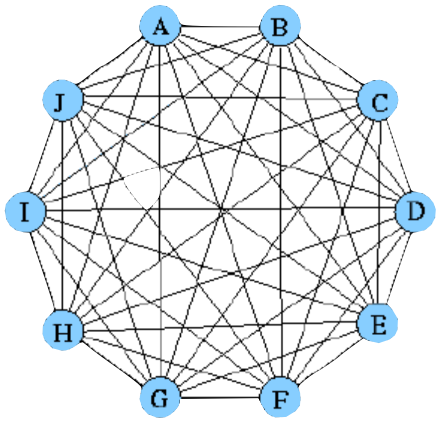
- **Las Vegas algorithms**
  - always correct; expected running time ("probably fast")
  - Examples:
    - randomized Quicksort,
    - randomized algorithm for closest pair
- **Monte Carlo algorithms (mostly correct):**
  - probably correct; guaranteed running time
  - Example: randomized primality test

Application: cryptosystems

Traditional encryption of messages with secret keys

Disadvantages:

1. **The key *k* has to be exchanged between A and B before the transmission of the message.**
2. **For messages between *n* parties *n*(*n*-1)/2 keys are required.**



Advantage:

Encryption and decryption can be computed very efficiently.

Duties of security providers

Guarantee…

- confidential transmission
- integrity of data
- authenticity of the sender
- reliable transmission

Public-key cryptosystems

Diffie and Hellman (1976)

Idea: Each participant A has two keys:

1. **a public key P_A accessible to every other participant**

2. **a private (or: secret) key S_A only known to A.**

Public-key cryptosystems

D = **set of all legal messages,**

e.g. the set of all bit strings of finite length

$$P_A(\ ),S_A(\ ):D\xrightarrow{1-1}D$$

Three conditions:

1. P_A **and** S_A **can be computed efficiently**

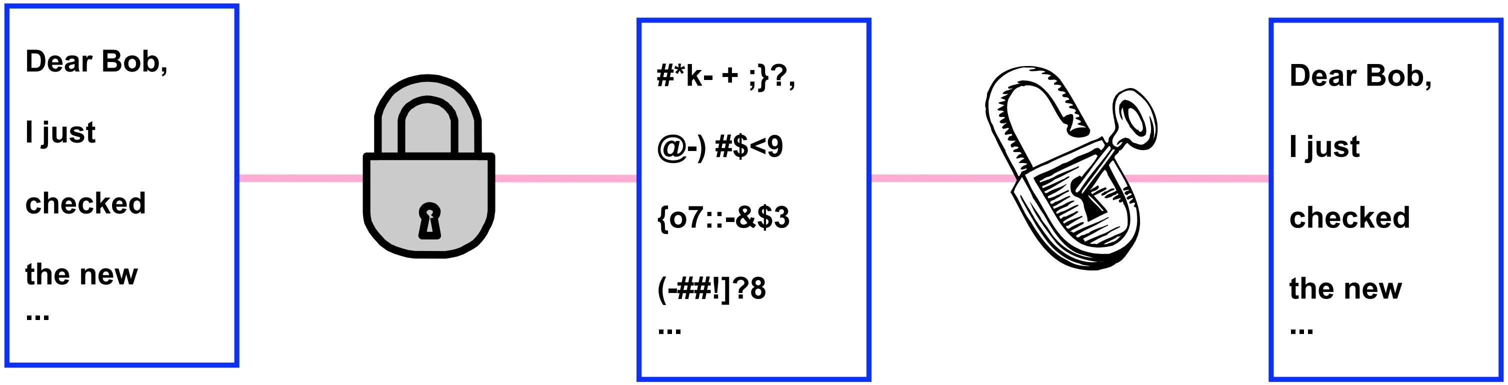
2. $S_A(P_A(M)) = M$ **and** $P_A(S_A(M)) = M$

(P_A, S_A **are inverse functions)**

3. S_A **cannot be computed from** P_A **(with reasonable effort)**

Encryption in a public-key system

A sends a message *M* to *B*.



Encryption in a public-key system

1. **A accesses B's public key P_B (from a public directory or directly from B).**
2. **A computes the encrypted message $C = P_B(M)$ and sends** *C to* **B***.*
3. **After B has received message C, B decrypts the message with his own private key S_B: $M = S_B(C)$**

Generating a digital signature

A sends a digitally signed message M' to B:

1. **A computes the digital signature σ for M' with her own private key:**

$$\sigma = S_A(M')$$

2. **A sends the pair (M',σ) to B.**

3. **After receiving (M',σ), B verifies the digital signature:**

$$P_A(\sigma) = M'$$

σ can by verified by anybody via the public P_A.

RSA cryptosystems

R. Rivest, A. Shamir, L. Adleman

Generating the public and private keys:

1. **Randomly select two primes p and q of similar size, each with $l+1$ bits ($l \geq 500$).**
2. **Let $n = p \cdot q$**
3. **Let e be an integer that does not divide $(p - 1) \cdot (q - 1)$.**
4. **Calculate $d = e^{-1} \bmod (p - 1)(q - 1)$**

   i.e.: $d \cdot e \equiv 1 \bmod (p - 1)(q - 1)$

RSA cryptosystems

5. Publish $P = (e, n)$ as public key

6. Keep $S = (d, n)$ as private key

Divide message (represented in binary) in blocks of size $2 \cdot l$.

Interpret each block M as a binary number: $0 \leq M < 2^{2 \cdot l}$

$$P(M) = M^e \bmod n \qquad\qquad S(C) = C^d \bmod n$$

Multiplicative Inverse

- **Theorem (GCD recursion theorem)**
- For any numbers a and b with b>0
  GCD(a,b) = GCD(b, a mod b)
- **Algorithm Euclid**
  Input: Two integers a and b with b ≥ 0
  Output: GCD(a,b)

```
if b=0
    then return a
    else return Euclid(b, a mod b)
```

Multiplicative Inverse

- **Algorithm Extended-Euclid**

  Input: Two integers a and b with b ≥ 0

  Output: GCD(a,b) and two integers x and y with

  xa+yb=GCD(a,b)

  if b=0 then return (a,1,0)

  else (d,x‘,y‘) := Extended-Euclid(b,a mod b)

  x:= y‘; y= $x' - \lfloor a/b \rfloor y'$;

  return (d,x,y)

- **Application:** a=(p-1)(q-1), b= u

  The algorithm returns numbers x and y with

  x(p-1)(q-1) + y e = GCD((p-1)(q-1),e) =1

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