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Exercise No. 11
Algorithms and Methods for Distributed Storage
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Exercise 11 *Allocation and Linear Programming*
Consider the following servers

- s_1 with size 500 GB and bandwidth 100 MB/s
- s_2 with size 100 GB and bandwidth 50 MB/s
- s_3 with size 1 GB and bandwidth 1000 MB/s

and the following documents

- d_1 with size 100 GB and popularity $\frac{1}{111}$
- d_2 with size 5 GB and popularity $\frac{100}{111}$
- d_3 with size 100 GB and popularity $\frac{10}{111}$

Determine the matrix A and vectors b and c for the linear programming problem which describes the optimal solution with respect to

1. **AvSeqTime** and
2. **AvParTime**.

Solution for AvSeqTime:

$$x = \begin{pmatrix} A_{1,1} \\ A_{1,2} \\ A_{1,3} \\ A_{2,1} \\ A_{2,2} \\ A_{2,3} \\ A_{3,1} \\ A_{3,2} \\ A_{3,3} \\ v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

$A_{d,s}$: share of document d on server s
v_i: auxiliary variables to transform $\sum_d A_{d,s} \leq |s|$ into $\sum_d A_{d,s} + v_d = |s|$

Constraints:

1. $\sum_d A_{d,s} \leq |s|$
2. $\sum_s A_{d,s} = |d|$

$$A = \left(\begin{array}{ccccccccc|ccc} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ \hline 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \qquad b = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} 100 \\ 5 \\ 100 \\ 500 \\ 100 \\ 1 \end{pmatrix}$$

$x \geq 0$, $Ax = b$, Minimize $c^T x$, where

$$c = \begin{pmatrix} b(1) \cdot p(1) \\ b(2) \cdot p(1) \\ b(3) \cdot p(1) \\ b(1) \cdot p(2) \\ b(2) \cdot p(2) \\ b(3) \cdot p(2) \\ b(1) \cdot p(3) \\ b(2) \cdot p(3) \\ b(3) \cdot p(3) \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 100 \cdot \frac{1}{111} \\ 50 \cdot \frac{1}{111} \\ 1000 \cdot \frac{1}{111} \\ 100 \cdot \frac{100}{111} \\ 50 \cdot \frac{100}{111} \\ 1000 \cdot \frac{100}{111} \\ 100 \cdot \frac{10}{111} \\ 50 \cdot \frac{10}{111} \\ 1000 \cdot \frac{10}{111} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Solution for AvParTime:

$$x = \begin{pmatrix} A_{1,1} \\ A_{1,2} \\ A_{1,3} \\ A_{2,1} \\ A_{2,2} \\ A_{2,3} \\ A_{3,1} \\ A_{3,2} \\ A_{3,3} \\ v_1 \\ v_2 \\ v_3 \\ m_1 \\ m_2 \\ m_3 \\ w_{1,1} \\ w_{1,2} \\ w_{1,3} \\ w_{2,1} \\ w_{2,2} \\ w_{2,3} \\ w_{3,1} \\ w_{3,2} \\ w_{3,3} \end{pmatrix}$$

$m_d = \max_s \frac{A_{d,s}}{b(s)}$ (maximum access time)
$v_i, w_{i,j}$: auxiliary variables

Constraints:

1. $\sum_d A_{d,s} \leq |s|$
2. $\sum_s A_{d,s} = |d|$

Additional constraints: $\forall s : m_d \geq \frac{A_{d,s}}{b(s)}$
This is expressed through $\frac{A_{d,s}}{b(s)} - m_d + w_{d,s} = 0$

$$b = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ s_1 \\ s_2 \\ s_3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 100 \\ 5 \\ 100 \\ 500 \\ 100 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$A = \left(\begin{array}{ccccccccc|ccc|ccc|ccccccccc}
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
\frac{1}{100} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{50} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{1000} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{100} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{50} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{1000} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{100} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{50} & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{1000} & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array} \right)$$

$x \geq 0$, $Ax = b$, Minimize $c^T x$, where

$$c = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{111} \\ \frac{100}{111} \\ \frac{10}{111} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$