ALBERT-LUDWIGS-UNIVERSITÄT FREIBURG

# Algorithms and Methods for Distributed Storage Networks

## 5 Raid-6 Encoding

**Christian Schindelhauer**

Albert-Ludwigs-Universität Freiburg
Institut für Informatik
Rechnernetze und Telematik
Wintersemester 2007/08

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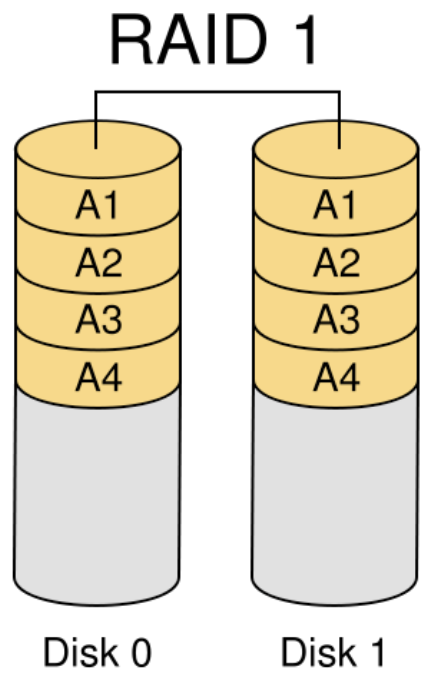
# RAID

- **Redundant Array of Independent Disks**
  - Patterson, Gibson, Katz, „A Case for Redundant Array of Inexpensive Disks“, 1987
- **Motivation**
  - Redundancy
    - error correction and fault tolerance
  - Performance (transfer rates)
  - Large logical volumes
  - Exchange of hard disks, increase of storage during operation
  - Cost reduction by use of inexpensive hard disks

# Raid 1

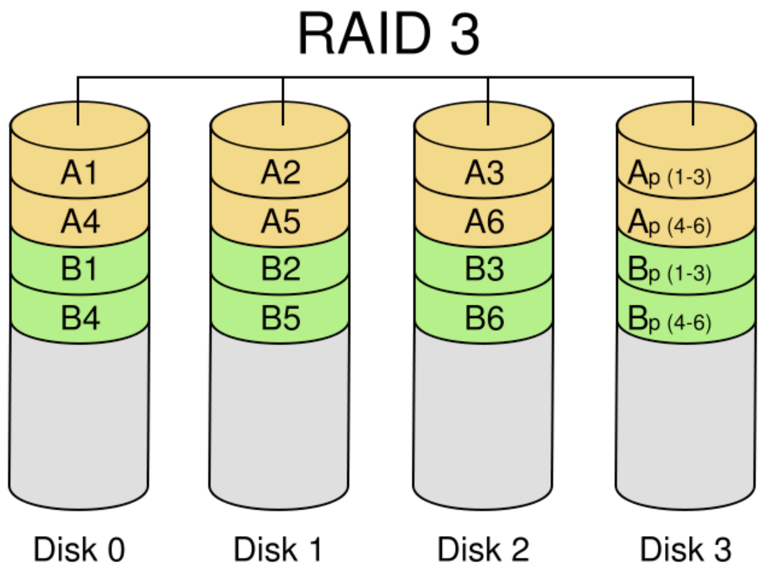
- **Mirrored set without parity**
  - Fragments are stored on all disks
- **Performance**
  - if multi-threaded operating system allows split seeks then
  - faster read performance
  - write performance slightly reduced
- **Error correction or redundancy**
  - all but one hard disks can fail without any data damage
- **Capacity reduced by factor 2**




http://en.wikipedia.org/wiki/RAID

# Raid 3

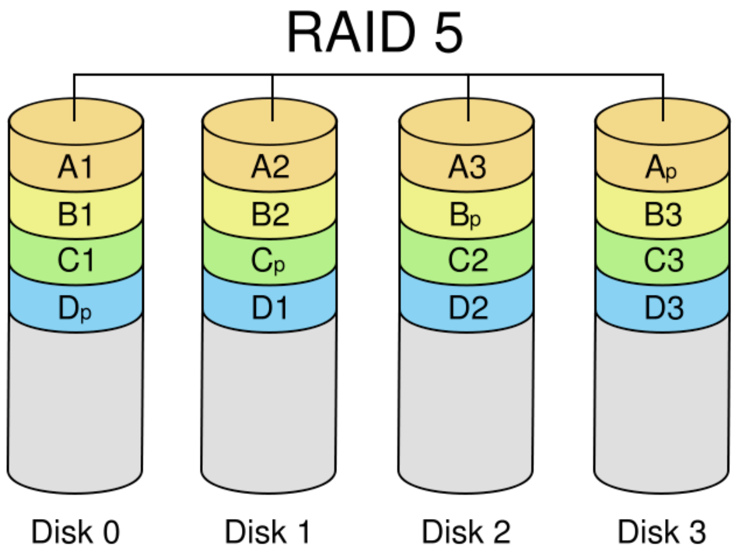
- **Striped set with dedicated parity (byte level parity)**
  - Fragments are distributed on all but one disks
  - One dedicated disk stores a parity of corresponding fragments of the other disks
- **Performance**
  - improved read performance
  - write performance reduced by bottleneck parity disk
- **Error correction or redundancy**
  - one hard disks can fail without any data damage
- **Capacity reduced by 1/n**




http://en.wikipedia.org/wiki/RAID

# Raid 5

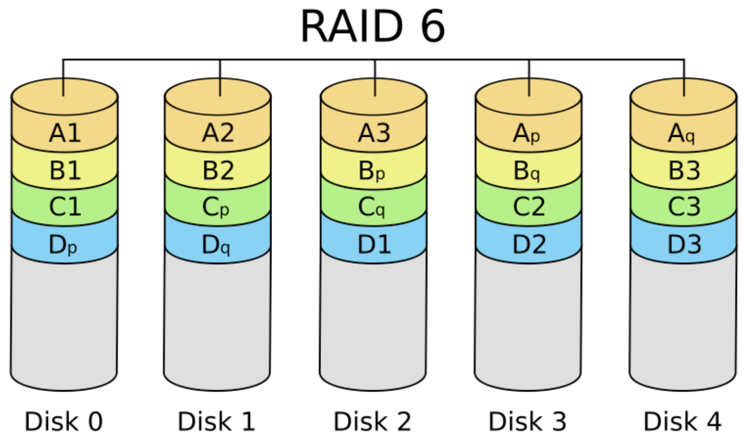
- **Striped set with distributed parity (interleave parity)**
  - Fragments are distributed on all but one disks
  - Parity blocks are distributed over all disks
- **Performance**
  - improved read performance
  - improved write performance
- **Error correction or redundancy**
  - one hard disks can fail without any data damage
- **Capacity reduced by 1/n**




http://en.wikipedia.org/wiki/RAID

# Raid 6

- **Striped set with dual distributed parity**
  - Fragments are distributed on all but two disks
  - Parity blocks are distributed over two of the disks
    - one uses XOR other alternative method
- **Performance**
  - improved read performance
  - improved write performance
- **Error correction or redundancy**
  - two hard disks can fail without any data damage
- **Capacity reduced by 2/n**

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Algorithms and Methods for
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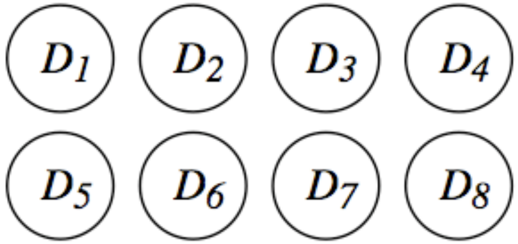
# RAID 6 - Encodings

# Literature

- **A Tutorial on Reed-Solomon Coding for Fault-Tolerance in RAID-like Systems, James S. Plank , 1999**
- **The RAID-6 Liberation Codes, James S. Plank, FAST ´08, 2008**

# Principle of RAID 6

- **Data units $D_1$, ..., $D_n$**
  - w: size of words
    - w=1 bits,
    - w=8 bytes, ...
- **Checksum devices $C_1$,$C_2$,..., $C_m$**
  - computed by functions $C_i$=Fi($D_1$,...,$D_n$)
- **Any n words from data words and check words**
  - can decode all n data units




$$C_1 = F_1(D_1, D_2, D_3, D_4, D_5, D_6, D_7, D_8)$$

$$C_2 = F_2(D_1, D_2, D_3, D_4, D_5, D_6, D_7, D_8)$$

A Tutorial on Reed-Solomon Coding for Fault-Tolerance in RAID-like Systems, James S. Plank , 1999

# Principle of RAID 6

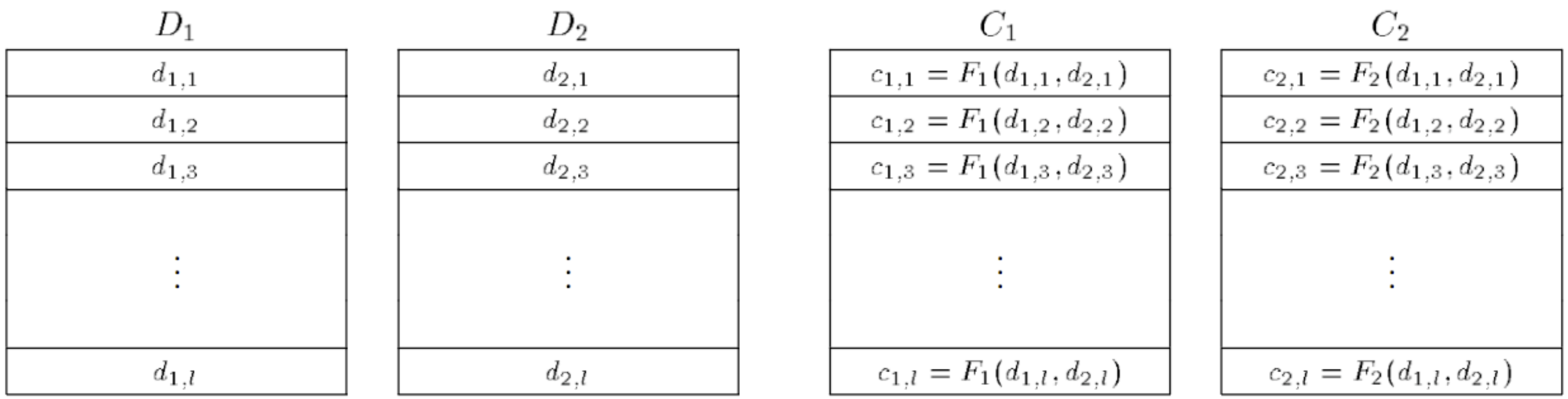



Figure 2: Breaking the storage devices into words ($n = 2, m = 2, l = \frac{8k}{w}$)

A Tutorial on Reed-Solomon Coding for Fault-Tolerance in RAID-like Systems, James S. Plank , 1999

# Operations

- **Encoding**
  - Given new data elements, calculate the check sums
- **Modification (update penalty)**
  - Recompute the checksums (relevant parts) if one data element is modified
- **Decoding**
  - Recalculate lost data after one or two failures
- **Efficiency**
  - speed of operations
  - check disk overhead
  - ease of implementation and transparency

RAID 6 Encodings

# Reed-Solomon

# Vandermonde-Matrix

$$\begin{bmatrix} f_{1,1} & f_{1,2} & \dots & f_{1,n} \\ f_{2,1} & f_{2,2} & \dots & f_{2,n} \\ \vdots & \vdots & & \vdots \\ f_{m,1} & f_{m,2} & \dots & f_{m,n} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & 2 & 3 & \dots & n \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & 2^{m-1} & 3^{m-1} & \dots & n^{m-1} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_m \end{bmatrix}$$

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# Complete Matrix

$$
\begin{bmatrix}
1 & 0 & 0 & \dots & 0 \\
0 & 1 & 0 & \dots & 0 \\
\vdots & \vdots & \vdots & & \vdots \\
0 & 0 & 0 & \dots & 1 \\
1 & 1 & 1 & \dots & 1 \\
1 & 2 & 3 & \dots & n \\
\vdots & \vdots & \vdots & & \vdots \\
1 & 2^{m-1} & 3^{m-1} & \dots & n^{m-1}
\end{bmatrix}
\begin{bmatrix}
d_1 \\ d_2 \\ \vdots \\ d_n
\end{bmatrix}
=
\begin{bmatrix}
d_1 \\ d_2 \\ \vdots \\ d_n \\ c_1 \\ c_2 \\ \vdots \\ c_m
\end{bmatrix}
$$

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# Galois Fields

- **$GF(2^w)$ = Finite Field over $2^w$ elements**
  - Elements are all binary strings of length w
  - $0 = 0^w$ is the neutral element for addition
  - $1 = 0^{w-1}1$ is the neutral element for multiplication
- **u + v = bit-wise Xor of the elements**
  - e.g. 0101 + 1100 = 1001
- **a b= product of polynomials modulo 2 and modulo an irreducible polynomial q**
  - i.e. $(a_{w-1} \ldots a_1\ a_0)\ (b_{w-1} \ldots b_1\ b_0) =$

$$((a_0 + a_1 x + \ldots + a_{w-1} x^{w-1})(b_0 + b_1 x + \ldots + b_{w-1} x^{w-1}) \bmod q(x)) \bmod 2)$$

# Example: $GF(2^2)$

| Generated Element of $GF(4)$ | Polynomial Element of $GF(4)$ | Binary Element $b$ of $GF(4)$ | Decimal Representation of $b$ |
|---|---|---|---|
| 0 | 0 | 00 | 0 |
| $x^0$ | 1 | 01 | 1 |
| $x^1$ | $x$ | 10 | 2 |
| $x^2$ | $x+1$ | 11 | 3 |

| + | 0 = 00 | 1 = 01 | 2 = 10 | 3 = 11 |
|---|---|---|---|---|
| 0 =00 | 0 | 1 | 2 | 3 |
| 1 =01 | 1 | 0 | 3 | 2 |
| 2 =10 | 2 | 3 | 0 | 1 |
| 3 =11 | 3 | 2 | 1 | 0 |

$q(x) = x^2+x+1$

| * | 0 = 0 | 1 = 1 | 2 = x | 3 = x+1 |
|---|---|---|---|---|
| 0 = 0 | 0 | 0 | 0 | 0 |
| 1 = 1 | 0 | 1 | 2 | 3 |
| 2 = x | 0 | 2 | 3 | 1 |
| 3 = x+1 | 0 | 3 | 1 | 2 |

$2 \cdot 3 = x(x+1) = x^2+x = 1 \bmod x^2+x+1 = 1$

$2 \cdot 2 = x^2 = x+1 \bmod x^2+x+1 = 3$

# Irreducible Polynomials

- **Irreducible polynomials cannot be factorized**
  - counter-example: $x^2+1 = (x+1)^2 \bmod 2$
- **Examples:**
  - w=2: $x^2+x+1$
  - w=4: $x^4+x+1$
  - w=8: $x^8+x^4+x^3+x^2+1$
  - w=16: $x^{16}+x^{12}+x^3+x+1$
  - w=32: $x^{32}+x^{22}+x^2+x+1$
  - w=64: $x^{64}+x^4+x^3+x+1$

# Fast Multiplication

- **Powers laws**
  - Consider: $\{2^0, 2^1, 2^2, \ldots\}$
  - $= \{x^0, x^1, x^2, x^3, \ldots$
  - = exp(0), exp(1), ...
- **exp(x+y) = exp(x) exp(y)**
- **Inverse: log(exp(x)) = x**
  - $\log(x \cdot y) = \log(x) + \log(y)$
- **x y = exp(log(x) + log(y))**
  - Warning: integer addition!!!
- **Use tables to compute exponential and logarithm function**

# Example: GF(16)

$q(x)= x^4+x+1$

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| exp(x) | 1 | $x$ | $x^2$ | $x^3$ | $1+x$ | $x+x^2$ | $x^2+x^3$ | $1+x+x^3$ | $1+x^2$ | $x+x^3$ | $1+x+x^2$ | $x+x^2+x^3$ | $1+x+x^2+x^3$ | $1+x^2+x^3$ | $1+x^3$ | 1 |
| exp(x) | 1 | 2 | 4 | 8 | 3 | 6 | 12 | 11 | 5 | 10 | 7 | 14 | 15 | 13 | 9 | 1 |

| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| log(x) | 0 | 1 | 4 | 2 | 8 | 5 | 10 | 3 | 14 | 9 | 7 | 6 | 13 | 11 | 12 |

- 5 · 12 = exp(log(5)+log(12)) = exp(8+6) = exp(14) = 9
- 7 · 9 = exp(log(7)+log(9)) = exp(10+14) = exp(24) = exp(24-15) = exp(9) = 10

# Example: Reed Solomon for GF[$2^4$]

- **Compute carry bits for three hard disks by computing**

$$F = \begin{bmatrix} 1^0 & 2^0 & 3^0 \\ 1^1 & 2^1 & 3^1 \\ 1^2 & 2^2 & 3^2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 5 \end{bmatrix}$$

- **F D = C**
  - where D is the vector of three data words
  - C is the vector of the three parity words
- **Store D and C on the disks**

# Complexity of Reed-Solomon

- **Encoding**
  - Time: O(k n) $GF[2^w]$-operations for k check words and n disks
- **Modification**
  - like Encoding
- **Decoding**
  - Time: $O(n^3)$ for matrix inversion
- **Ease of implementation**
  - check disk overhead is minimal
  - complicated decoding

# Cauchy-Reed-Solomon

- **An XOR-Based Erasure-Resilient Coding Scheme, Blömer, Kalfane, Karp, Karpinski, Luby, Zuckerman, 1995**

**Definition 5.1** *Let $F$ be a field and let $\{x_1, \ldots, x_m\}$, $\{y_1, \ldots, y_n\}$ be two sets of elements in $F$ such that*

*(i)* $\forall i \in \{1, \ldots, m\} \, \forall j \in \{1, \ldots, n\}: \quad x_i + y_j \neq 0.$

*(ii)* $\forall i, j \in \{1, \ldots, m\}, i \neq j: \quad x_i \neq x_j$ *and* $\forall i, j \in \{1, \ldots, n\}, i \neq j: \quad y_i \neq y_j.$

*The matrix*

$$\begin{bmatrix} \frac{1}{x_1+y_1} & \frac{1}{x_1+y_2} & \cdots & \frac{1}{x_1+y_n} \\ \frac{1}{x_2+y_1} & \frac{1}{x_2+y_2} & \cdots & \frac{1}{x_2+y_n} \\ & & \ddots & \\ \frac{1}{x_{m-1}+y_1} & \frac{1}{x_{m-1}+y_2} & \cdots & \frac{1}{x_{m-1}+y_n} \\ \frac{1}{x_m+y_1} & \frac{1}{x_m+y_2} & \cdots & \frac{1}{x_m+y_n} \end{bmatrix}$$

*is called a Cauchy matrix over $F$.*

**Theorem 5.3** *The inverse of an $(n \times n)$-Cauchy matrix over a field $F$ can be computed using $\mathcal{O}(n^2)$ arithmetic operations in $F$.*

# Complexity of Cauchy-Reed-Solomon

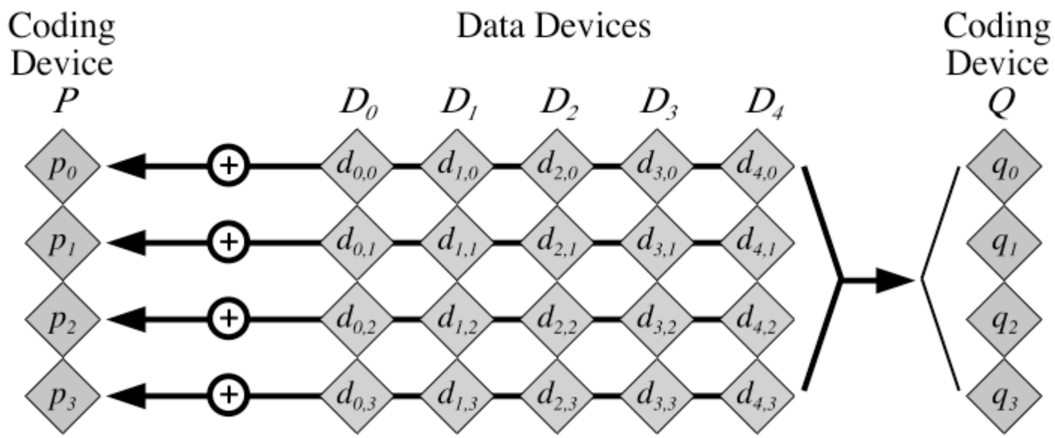
- **Encoding**
  - Time: O(k n) $GF[2^w]$-operations for k check words and n disks
- **Modification**
  - like Encoding
- **Decoding**
  - Time: $O(n^2)$ for matrix inversion
- **Ease of implementation**
  - check disk overhead is minimal
  - less complicated decoding, still not transparent

RAID 6 Encodings

# Parity Arrays

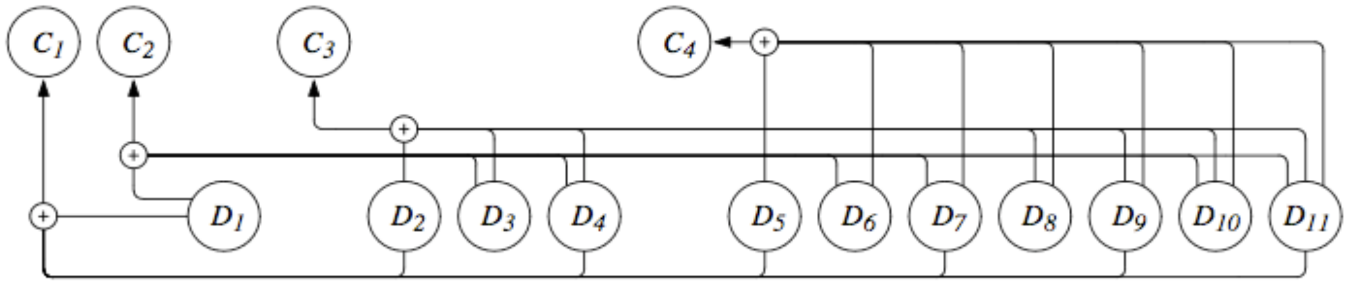
# Parity Arrays

- **Uses Parity of data bits**
- **Each check bit collects different subset of data bits**
- **Examples**
  - Evenodd
  - RDP



# Hamming Code

- **Use adapted version of Hamming code to compute check bits**
- **Problem: not flexible encoding for various number of disks or check codes**

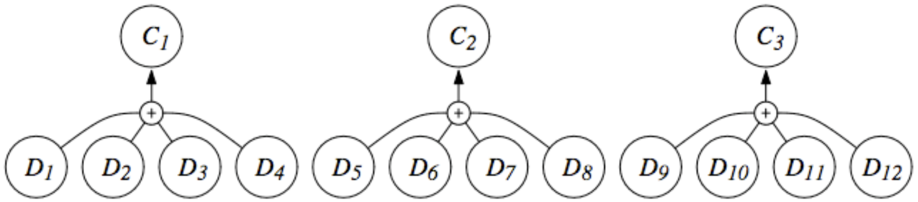



Hamming code, $n = 11, m = 4$

The RAID-6 Liberation Codes
James S. Plank

# One-Dimensional Parity

- **Organize data bits as n/m groups**
  - compute parity for each group
- **Results in m check bits**
- **Fast and simple computation for**
  - Coding, Decoding, Modification
- **Problem**
  - tolerates not all combinations of failures
  - unsafe solution for combined failure of check disk and data disk

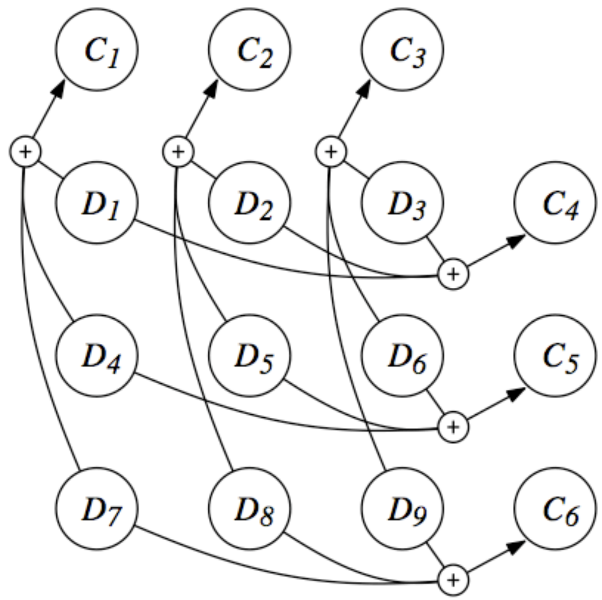



One-dimensional parity, $n = 12, m = 3$

A Tutorial on Reed-Solomon Coding for Fault-Tolerance in RAID-like Systems, James S. Plank , 1999

# Two-Dimensional Parity

- **Organize data disks as a k*k-square**
  - compute k parities for all rows
  - compute k parities for all columns
- **Results in 2k check bits**
- **Fast computation for**
  - Coding, Decoding, Modification
- **Safety**
  - tolerate only all combinations for two failures
  - tolerates not all combinations for three failures
- **Problem**
  - large number of hard disks
  - check disk overhead

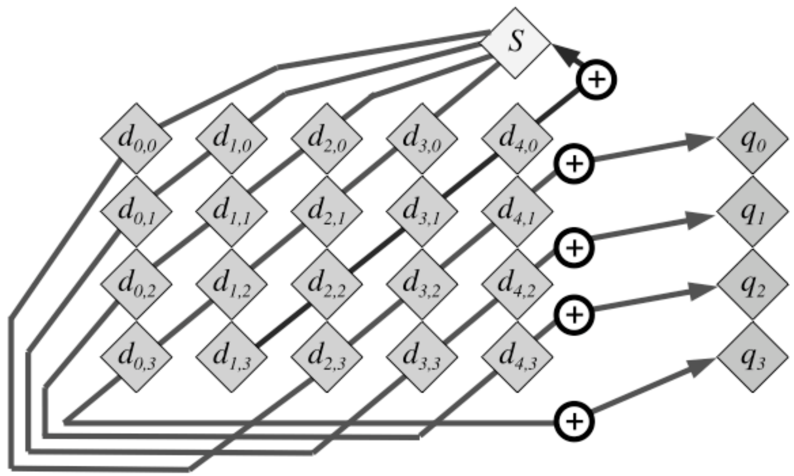



Two-dimensional parity, $n = 9, m = 6$

A Tutorial on Reed-Solomon Coding for Fault-Tolerance in RAID-like Systems, James S. Plank , 1999

# EVENODD-Encoding

- **Computes exactly two check works**
- **P = parity check word**
- **Q = parity over the diagonal elements**
- **Fast Encoding**
- **Decoding**
  - $O(n^2)$ time for n disks and n data bits
- **Optimal check disk overhead**
- **Generalized versions**
  - STAR code (Huang, Xu, FAST'05)
  - Feng, Deng, Bao, Shen, 2005

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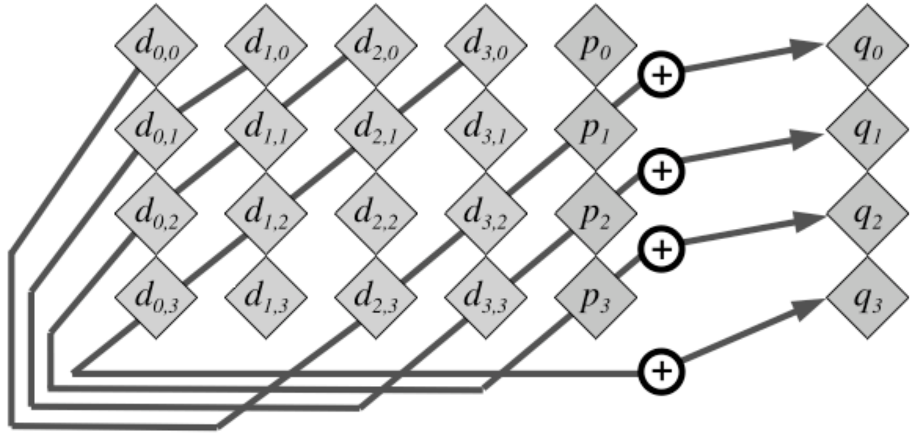
# RDP Coding

- **Row Diagonal Parity**
  - improved version of EVENODD
- **Two check words**
  - Parity over words
  - Use diagonal parities
- **Easier code**
- **Creates only two check words**

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# Liberation Codes

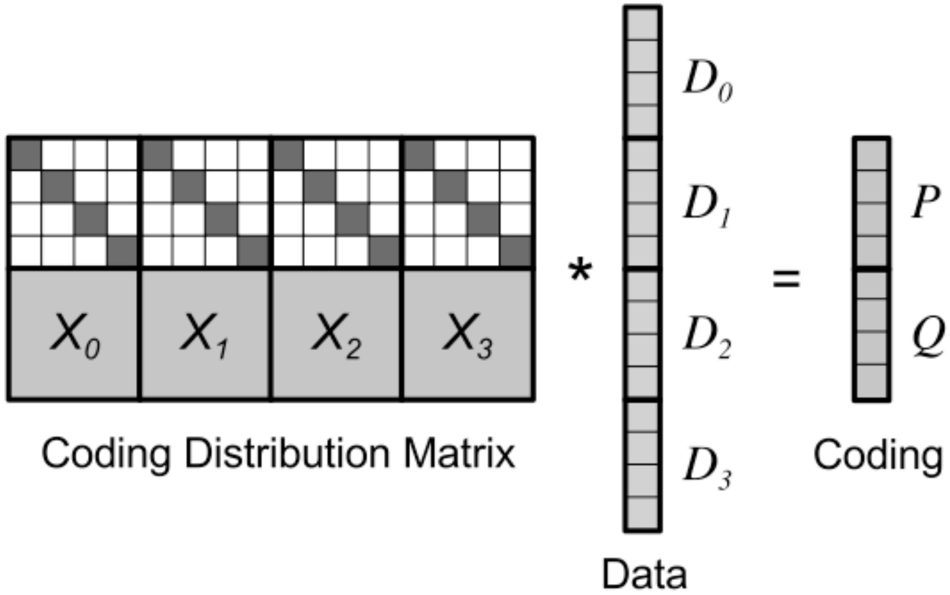


Figure 7: Bit matrix representation of RAID-6 coding when $k = 4$ and $w = 4$.

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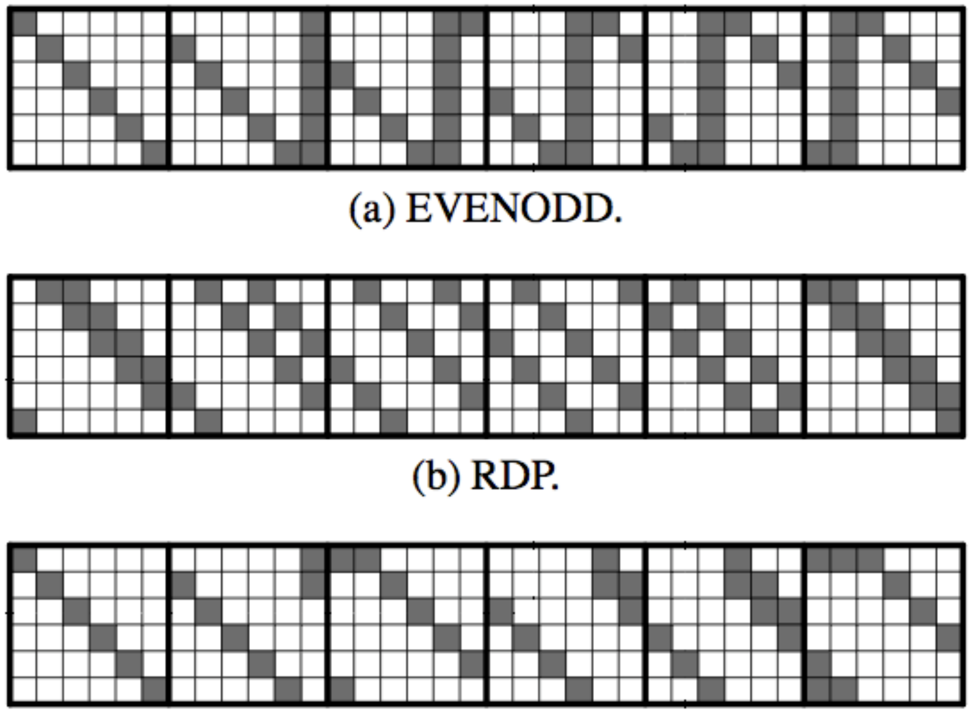
# Liberation Codes



(a) EVENODD.

(b) RDP.

(c) Cauchy Reed-Solomon coding.

Figure 8: The $X_i$ matrices defining the BDM's for various RAID-6 coding techniques, $k = 6$ and $w = 6$.

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# Liberation Codes

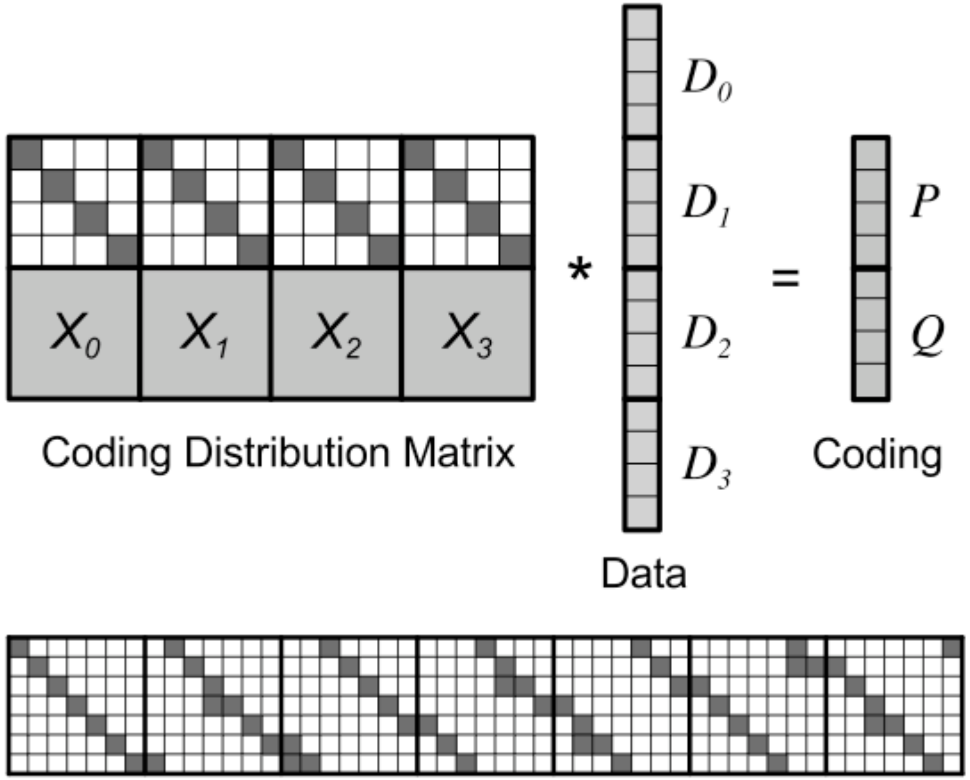


Figure 9: The $X_i$ matrices for the Liberation Code when $k = 7$ and $w = 7$.

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# Liberation Codes

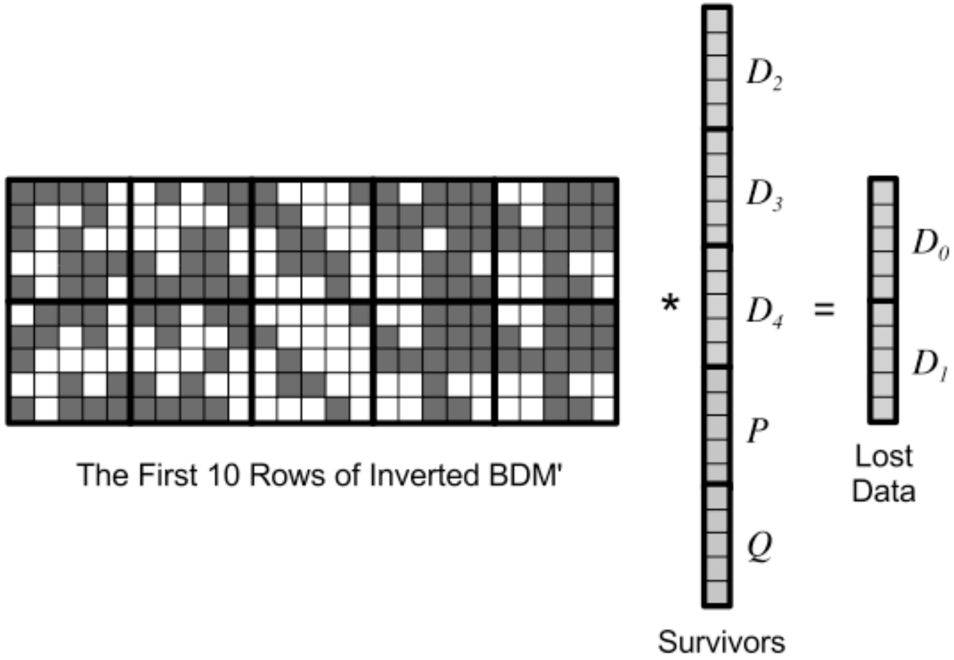


Figure 10: Decoding $D_0$ and $D_1$ from the Liberation Codes when $k = 5$ and $w = 5$.

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# Performance

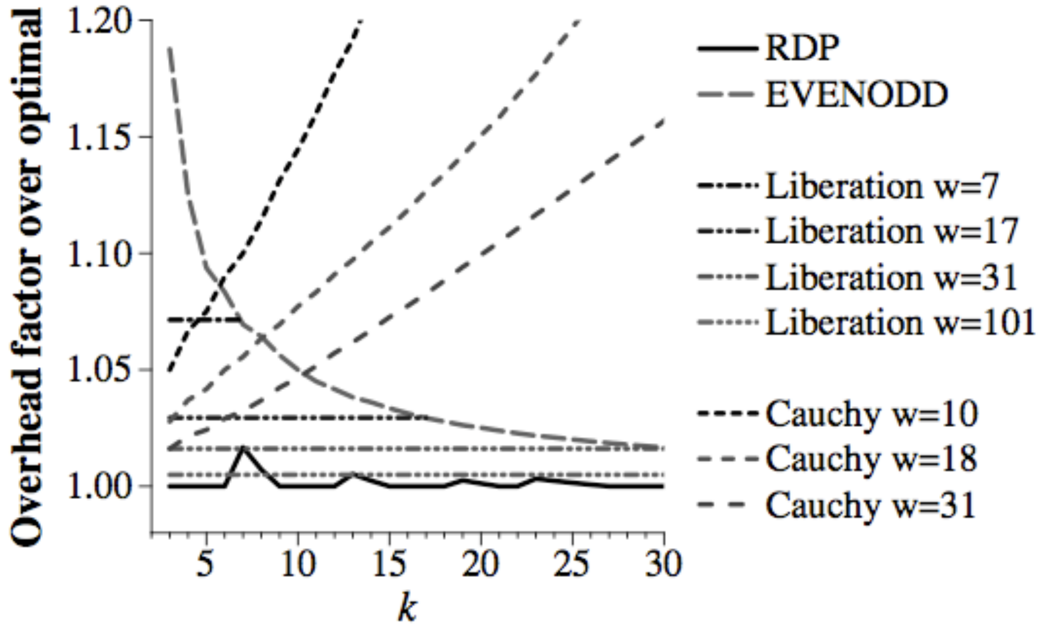


Figure 11: Encoding performance of various XOR-based RAID-6 techniques. Optimal encoding is $k-1$ XORs per coding word.

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# Performance

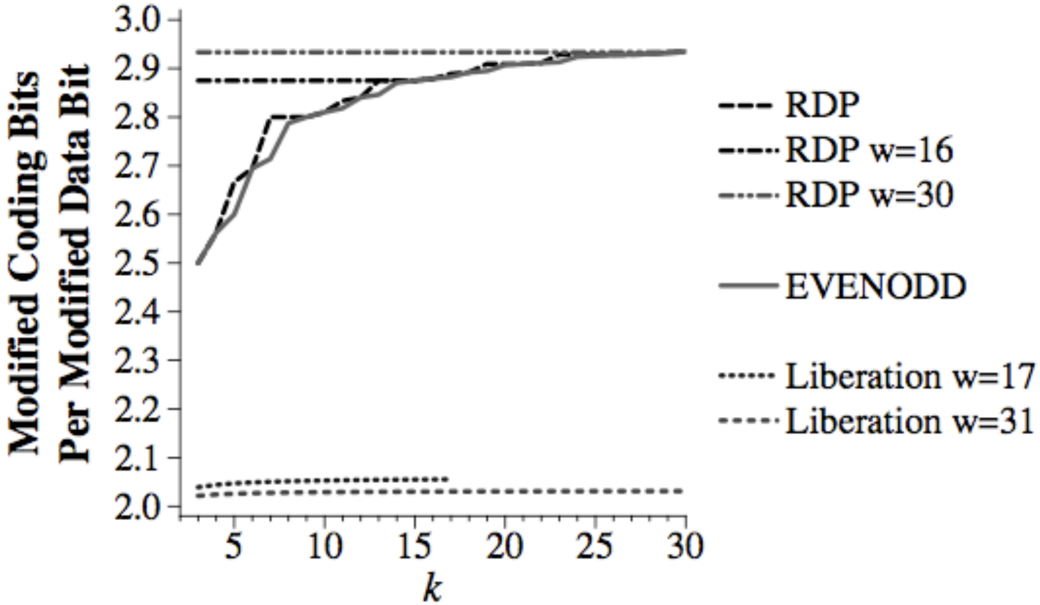


Figure 13: Modification performance of RDP, EVEN-ODD and Liberation codes.

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# Performance

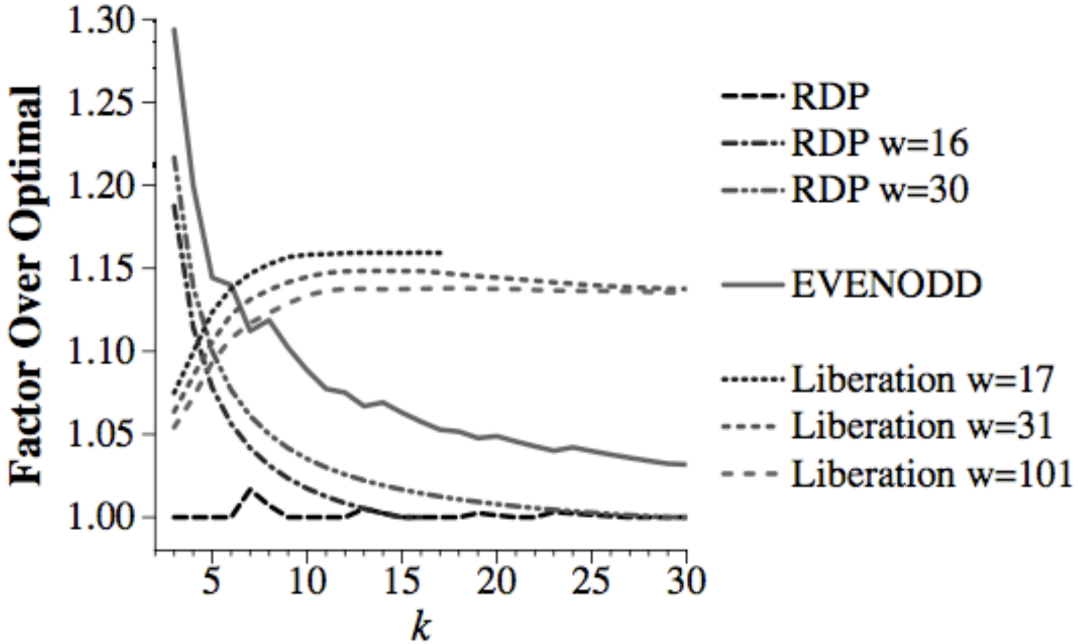


Figure 14: Decoding performance of RDP, EVENODD and Liberation codes.

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# Algorithms and Methods for Distributed Storage Networks

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**Christian Schindelhauer**

Albert-Ludwigs-Universität Freiburg
Institut für Informatik
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CoNe Freiburg

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