Peer-to-Peer Networks

Kelips
5th Week

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Summer 2008

Peer-to-Peer Networks

Kelips

Kelips

- **Indranil Gupta, Ken Birman, Prakash Linga, Al Demers, Robbert van Renesse**
  - Cornell University, Ithaca, New York
- **Kelip-kelip**
  - malay name for synchronizing fireflies
- **P2P Network**
  - uses DHT
  - constant lookup time
  - $O(n^{1/2})$ storage size
  - fast and robust update

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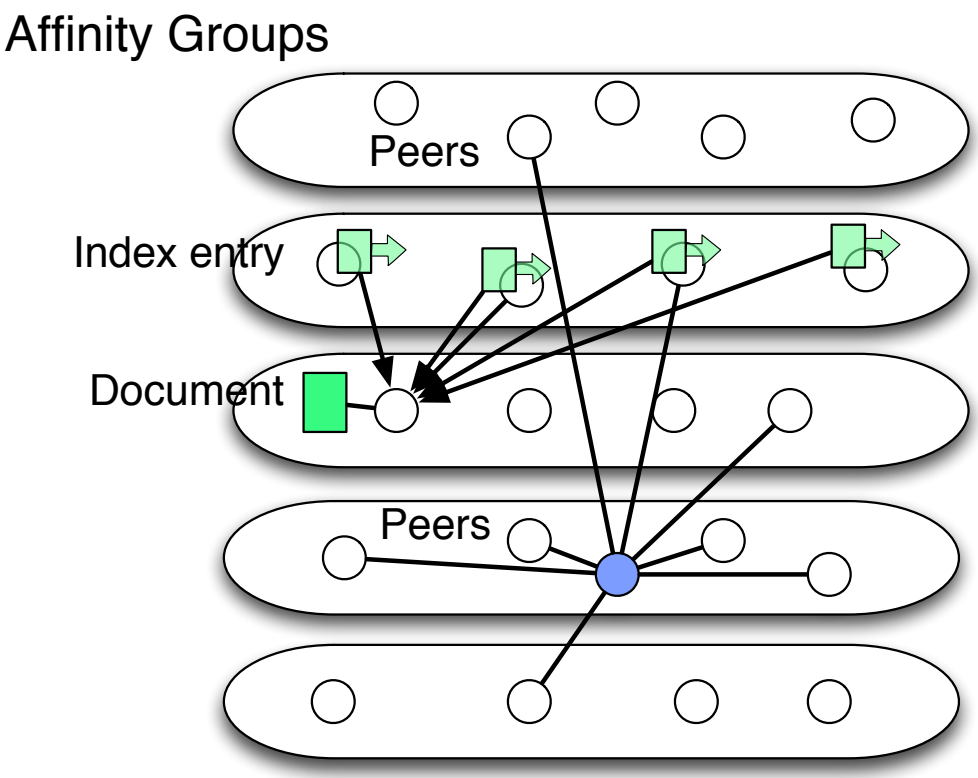
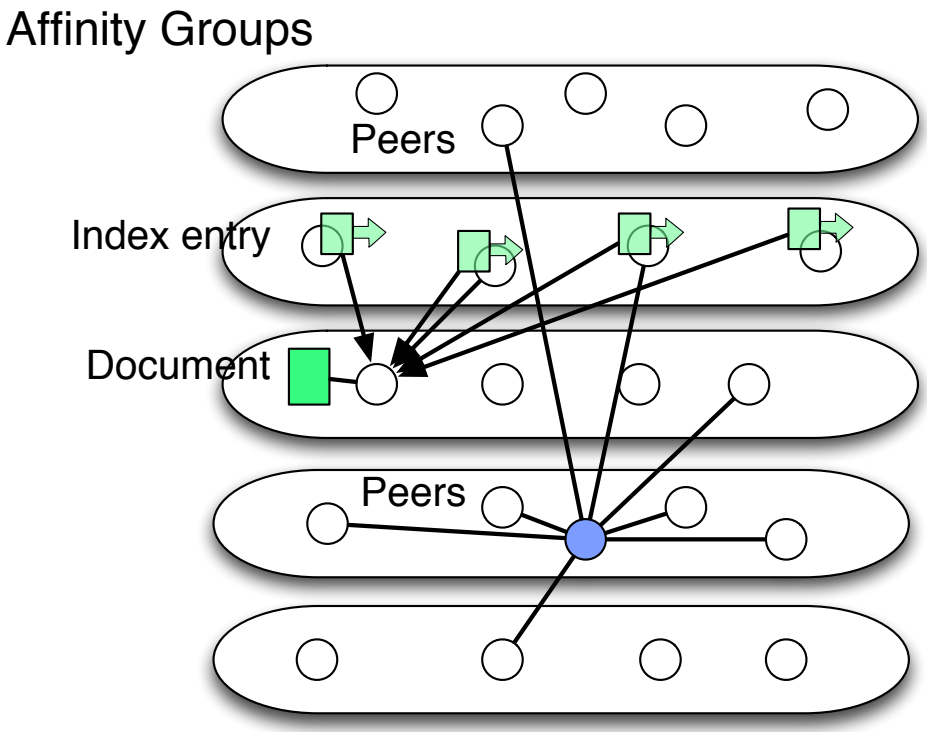
Kelips Overview

- **Peers are organized in k affinity groups**
  - peer position chosen by DHT mechanism
  - k is chosen as $n^{1/2}$ for n peers
- **Data is mapped to an affinity group using DHT**
  - all members of an affinity group store all data
- **Routing Table**
  - each peer knows all members of the affinity group
  - each peer knows at least one member of each affinity group
- **Updates**
  - are performed by epidemic algorithms



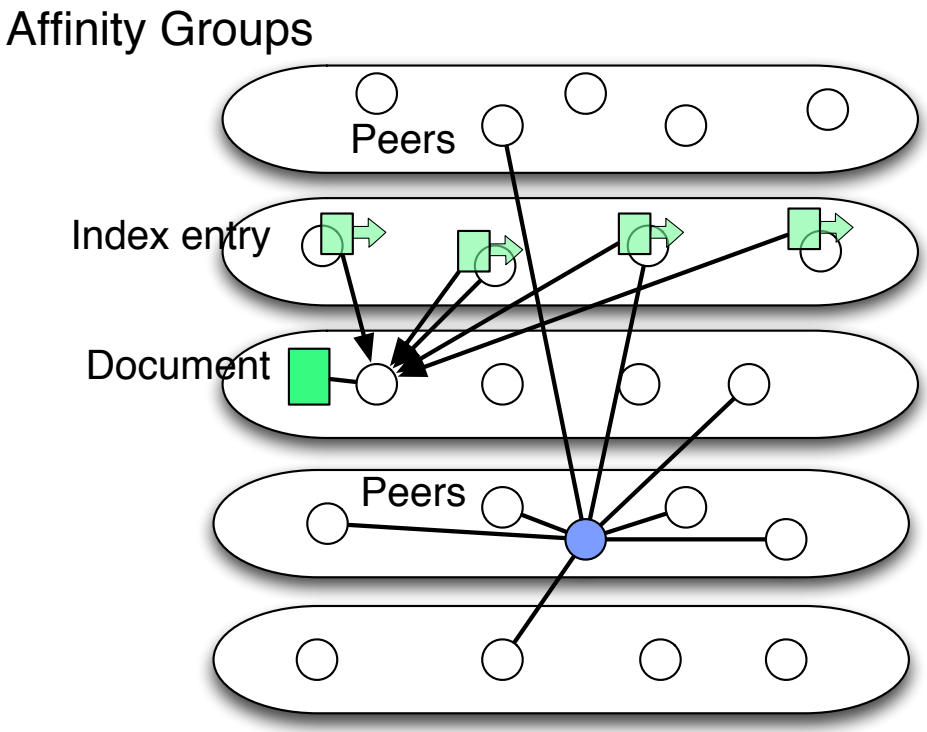
Routing Table

- **Affinity Group View**
  - Links to all O(n/k) group members
  - This set can be reduced to a partial set as long as the update mechanism works
- **Contacts**
  - For each of the other affinity group a small (constant-sized) set of nodes
  - O(k) links
- **Filetuples**
  - A (partial) set of tuples, each detailing a file name and host IP address of the node storing the file
  - O(F/k) entries, if F is the overall number of files
- **Memory Usage: O(n/k + k + F/k)**
  - for $\mathrm{k} = \mathrm{O}\left(\sqrt{n+F}\right)$ $\mathrm{O}\left(\sqrt{n+F}\right)$



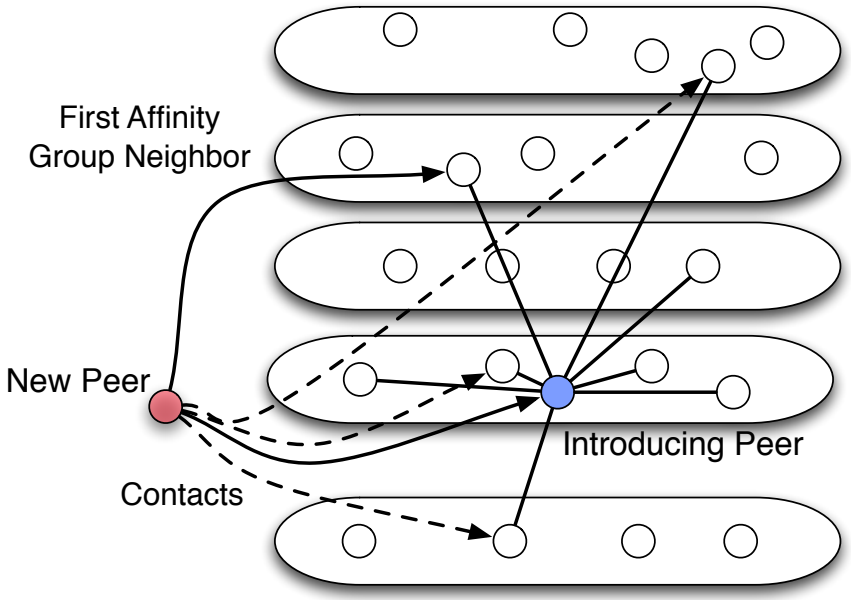
Lookup

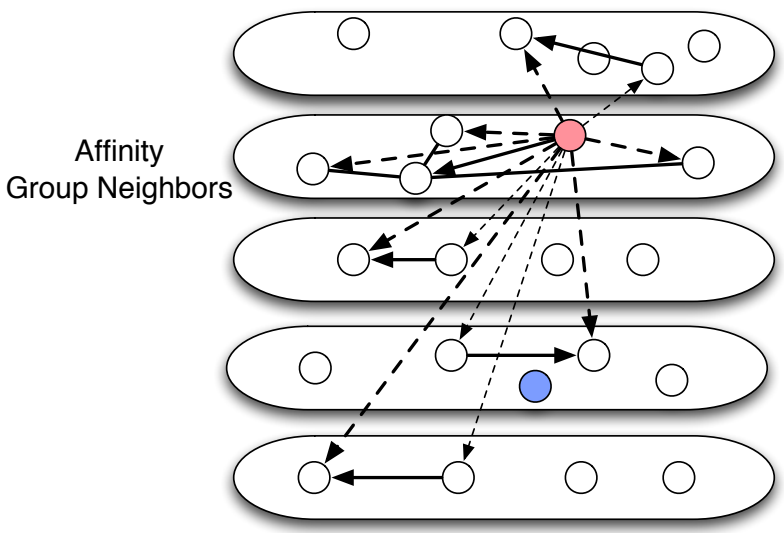
- **Lookup-Algorithm**
  - compute index value
  - find affinity group using hash function
  - contact peer from affinity group
  - receive index entry for file (if it exists)
  - contact peer with the document
- **Kelips needs four hops to retrieve a file**



Inserting a Peer

- **Algorithm**
  - Every new peer is introduced by a special peer, group or other method,
    - e.g. web-page, forum etc.
  - The new peer computes its affinity group and contacts any peer
  - The new peer asks for one contact of the affinity group and copies the contacts of the old affinity group
  - By contacting a neighbor node in the affinity group it receives all the necessary contacts and index filetuples
  - Every contact is replaced by a random replacement (suggested by the contact peer)
  - The peer starts an **epidemic algorithm** to update all links
- **Except the epidemic algorithm the runtime is O(k) and only O(k) messages are exchanged**





How to Add a Document

- **Start an Epidemic Algorithm to Spread the news in the affinity group**
- **Such an algorithm uses O(n/k) messages and needs O(log n) time**
- **We introduce Epidemic Algorithms later on**

How to Check Errors

- **Kelip works in heartbeats, i.e. discrete timing**
- **In every heartbeat each peer checks one neighbor**
- **If a neighbor does not answer for some time**
  - it is declared to be dead
  - this information is spread by an epidemic algorithm
- **Using the heartbeat mechanisms all nodes also refresh their neighbors**
- **Kelips quickly detects missing nodes and updates this information**

Discussion

- **Kelips has lookup time O(1), but needs $O(n^{1/2})$ sized Routing Table**
  - not counting the $O(F/n^{1/2})$ Filetuples
- **Chord, Pastry & Tapestry use lookup time O(log n) but only O(log n) memory units**
- **Kelips is a reasonable choice for medium sized networks**
  - up to some million peers and some hundred thousands index entries

To Do

- **What is an Epidemic Algorithm**

Peer-to-Peer Networks

Epidemic Algorithms

Epidemic Spread of Viruses

- **Observation**
  - most viruses do not prosper in real life
  - other viruses are very successful and spread fast
- **How fast do viruses spread?**
- **How many individuals of the popolation are infected?**
- **Problem**
  - social behavior and infection risk determine the spread
  - the reaction of a society to a virus changes the epidemy
  - viruses and individuals may change during the infection

Mathematical Models

- **SI-Model (rumor spreading)**
  - susceptible → infected
- **SIS-Model (birthrate/deathrate)**
  - susceptible → infected → susceptible
- **SIR-Model**
  - susceptible → infected → recovered

- **Continuos models**
  - deterministic
  - or stochastic
- **Lead to differential equations**

- **Discrete Models**
  - graph based models
  - random call based
- **Lead to the analysis of Markov Processes**

Infection Models

- **SI-Model (rumor spreading)**
  - susceptible → infected
  - At the beginning one individual is infected
  - Every contact infects another indiviual
  - In every time unit there are in the expectation ß contacts
- **SIS-Model (birthrate/deathrate)**
  - susceptible → infected → susceptible
  - similar as in the SI-Model, yet a share of δ of all infected individuals is healed and can receive the virus again
  - with probability δ an individual is susceptible again
- **SIR-Model**
  - susceptible → infected → recovered
  - like SI-Model, but healed individuals remain immune against the virus and do not transmit the virus again

SI-Model

- **Variables**
  - n: total number of individuals
    - remains constant
  - S(t): number of (healthy) susceptible individuals at time t
  - I(t): number of infected individuals
- **Relative shares**
  - s(t) := S(t)/n
  - i(t) := I(t)/n
- **At every time unit each individual contacts ß partners**
- **Assumptions:**
  - Among ß contact partnres ß s(t) are susceptible
  - All I(t) infected individuals infect ß s(t) I(t) other individuals in each round
- **Leads to the following recursive equations:**
  - $I(t+1) = I(t) + ß\ s(t)\ I(t)$
  - $i(t+1) = i(t) + ß\ i(t)\ s(t)$
  - $S(t+1) = S(t) - ß\ s(t)\ I(t)$
  - $s(t+1) = s(t) - ß\ i(t)\ s(t)$

SI-Model

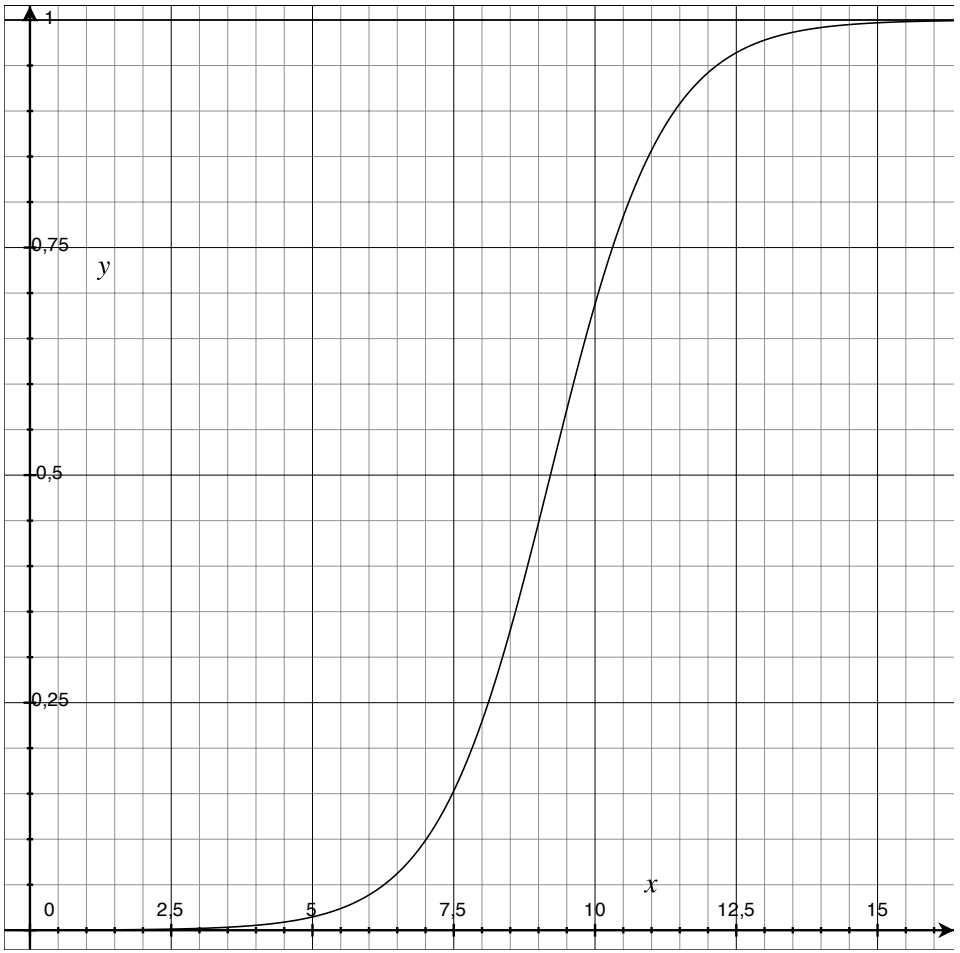
- i(t+1) = i(t) + ß i(t) s(t)
- s(t+1) = s(t) – ß i(t) s(t)
- Idea:
  - i(t) is a continuous function
  - i(t+1)-i(t) approximate first derivative $\frac{i(t+1)-i(t)}{1} \approx \frac{di(t)}{dt}$

$$\frac{di(t)}{dt} = \beta \cdot i(t)(1 - i(t))$$

- Solution: $$i(t) = \frac{1}{1 + \left(\frac{1}{i(0)} - 1\right) e^{-\beta t}}$$

SI-Model

- **The number of infected grows exponentially until half of all members are infected**
- **Then the number of susceptible decrease exponentially**



SIS-Model

- **Variables**
  - n: total number of individuals
    - remains constant
  - S(t): number of (healthy) susceptible individuals at time t
  - I(t): number of infected individuals
- **Relative shares**
  - s(t) := S(t)/n
  - i(t) := I(t)/n
- **At every time unit each individual contacts ß partners**

- **Assumptions:**
  - Among ß contact partnres ß s(t) are susceptible
  - All I(t) infected individuals infect ß s(t) I(t) other individuals in each round
  - A share of δ of all infected individuals is susceptible again
- **Leads to the following recursive equations:**
  - $I(t+1) = I(t) + ß\ i(t)\ S(t) - δ\ I(t)$
  - $i(t+1) = i(t) + ß\ i(t)\ s(t) - δ\ i(t)$
  - $S(t+1) = S(t) - ß\ i(t)\ S(t) + δ\ I(t)$
  - $s(t+1) = s(t) - ß\ i(t)\ s(t) + δ\ i(t)$

SI-Model

- **i(t+1) = i(t) + ß i(t) s(t) – δ i(t)**
- **s(t+1) = s(t) – ß i(t) s(t) + δ i(t)**
- **Idea:**
  - i(t) is a continuous function
  - i(t+1)-i(t) approximate first derivative

$$\frac{i(t+1) - i(t)}{1} \approx \frac{di(t)}{dt}$$

$$\frac{di(t)}{dt} = \beta \cdot i(t)(1 - i(t)) - \delta i(t)$$

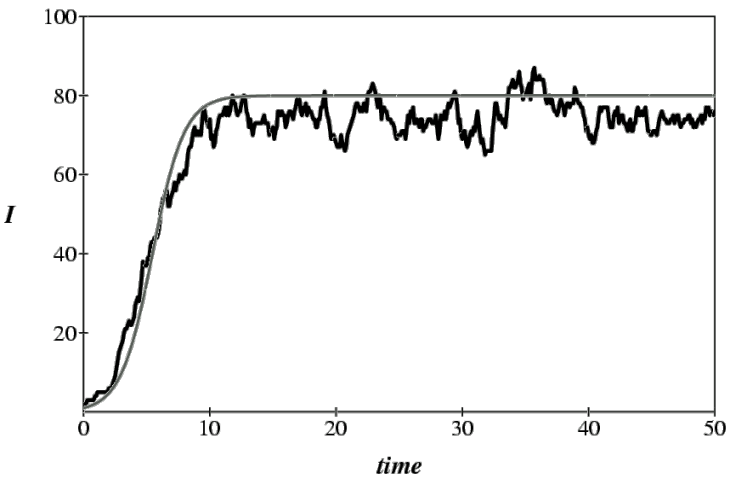
- **Solution:**
  - for $\rho = \frac{\delta}{\beta}$

$$i(t) = \frac{1 - \rho}{1 + \left(\frac{1-\rho}{i(0)} - 1\right) e^{-(\beta-\delta)t}}$$

SIS-Model
Interpretation of Solution

$$i(t) = \frac{1-\rho}{1+\left(\frac{1-\rho}{i(0)}-1\right)e^{-(\beta-\delta)t}} \qquad \rho = \frac{\delta}{\beta}$$

- **If ß < δ**
  - then i(t) is strictly decreasing
- **If ß > δ**
  - then i(t) converges against $1 - \rho = 1 - \delta/ß$
- **Same behavior in discrete model has been observed**
  - [Kephart,White‘94]



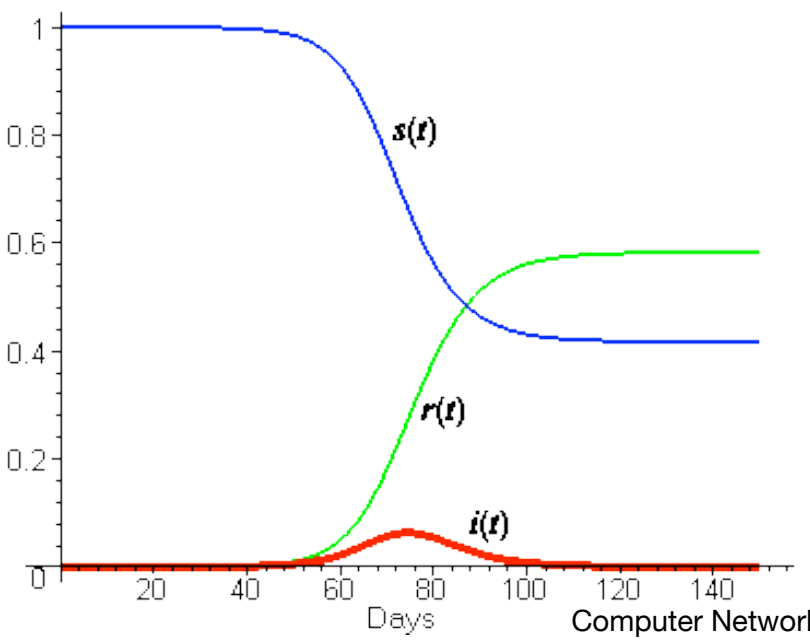
SIR-Model

- **Variables**
  - n: total number of individuals
    - remains constant
  - S(t): number of (healthy) susceptible individuals at time t
  - I(t): number of infected individuals
  - R(t): number or recovered individ.
- **Relative shares**
  - s(t) := S(t)/n
  - i(t) := I(t)/n
  - r(t) := R(t)/n
- **At every time unit each individual contacts ß partners**
- **Assumptions:**
  - Among ß contact partnres ß s(t) are susceptible
  - All I(t) infected individuals infect ß s(t) I(t) other individuals in each round
  - A share of δ of all infected individuals is immune (recovered) and never infected again
- **Leads to the following recursive equations:**
  - $I(t+1) = I(t) + ß\ i(t)\ S(t) - δ\ I(t)$
  - $i(t+1) = i(t) + ß\ i(t)\ i(t) - δ\ i(t)$
  - $S(t+1) = S(t) - ß\ i(t)\ S(t)$
  - $s(t+1) = s(t) - ß\ i(t)\ s(t)$
  - $R(t+1) = R(t) + δ\ I(t)$
  - $r(t+1) = r(t) + δ\ i(t)$

SIR-Model

- **The equations and its differential equations counterpart**
  - i(t+1) = i(t) + ß i(t) i(t) – δ i(t)
  - s(t+1) = s(t) – ß i(t) s(t)
  - r(t+1) = r(t) + δ i(t)
- **No closed solution known**
  - hence numeric solution
- **Example**
  - s(0) = 1
  - i(0) = 1,27 10-6
  - r(0) = 0
  - ß = 0,5
  - δ = 0,3333

$$\frac{ds(t)}{dt} = -\beta \cdot i(t)s(t)$$
$$\frac{di(t)}{dt} = \beta \cdot i(t)s(t) - \delta i(t)$$
$$\frac{dr(t)}{dt} = -\delta i(t)$$



Peer-to-Peer Networks

Epidemic Algorithms

Replicated Databases

- **Same data storage at all locations**
  - new entries appear locally
- **Data must be kept consistently**
- **Algorithm is supposed to be decentral and robust**
  - since connections and hosts are unreliable
- **Not all databases are known to all**
- **Solutions**
  - Unicast
    - New information is sent to all data servers
  - Problem:
    - not all data servers are known and can be reached
  - Anti-Entropy
    - Every local data server contacts another one and exchanges all information
    - total consistency check of all data
  - Problem
    - comunication overhead
- **Epicast …**

Epidemic Algorithms

- **Epicast**
  - new information is a rumor
  - as long the rumor is new it is distributed
  - Is the rumor old, it is known to all servers
- **Epidemic Algorithm [Demers et al 87]**
  - distributes information like a virus
  - robust alternative to BFS or flooding
- **Communication method**
  - Push & Pull, d.h. infection after $\log_3 n + O(\log \log n)$ rounds with high probability
- **Problem:**
  - growing number of infections increases comunication effort
  - trade-off between robustness and communication overhead

SI-Model for Graphs

- **Given a contact graph G=(V,E)**
  - n: number of nodes
  - I(t) := number of infected nodes in round t
  - i(t) = I(T)/n
  - S(t) := number of susceptible nodes in round t
    - I(t)+S(t)=n
  - s(t) = S(T)/n
- **Infection:**
- **If u is infected in round t and (u,v) $\in$ E, then v is infected in round t+1**

- **Graph determines epidemics**
- **Complete graph:**
  - 1 time unit until complete infection
- **Line graph**
  - n-1 time units until complete infection

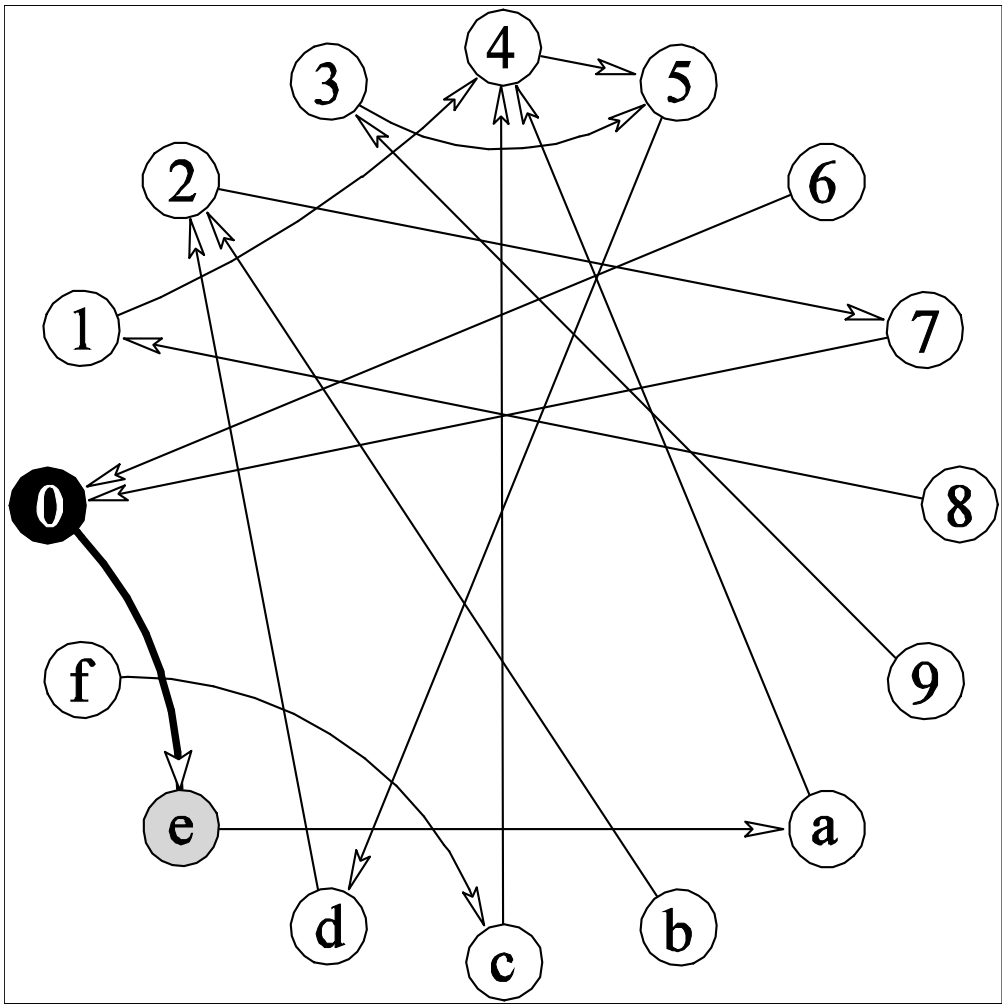
Epidemics in Static Random Graphs

- **Zufallsgraph $G_{n,p}$**
  - n nodes
  - Each directed edge occurs with independent probability p
- **Expected indegre $\gamma = p\,(n-1)$**
- **How fast does an epidemic spread in $G_{n,p}$, if $\gamma \in O(1)$?**
- **Observation für n>2:**
  - With probability $\geq 4^{-\gamma}$ and $\leq e^{-\gamma}$
    - a node has in-degree 0 and cannot be infected
    - a node has out-degree 0, and cannot infect others
- **Implications:**
  - Random (static) graph is not a suitable graph for epidemics

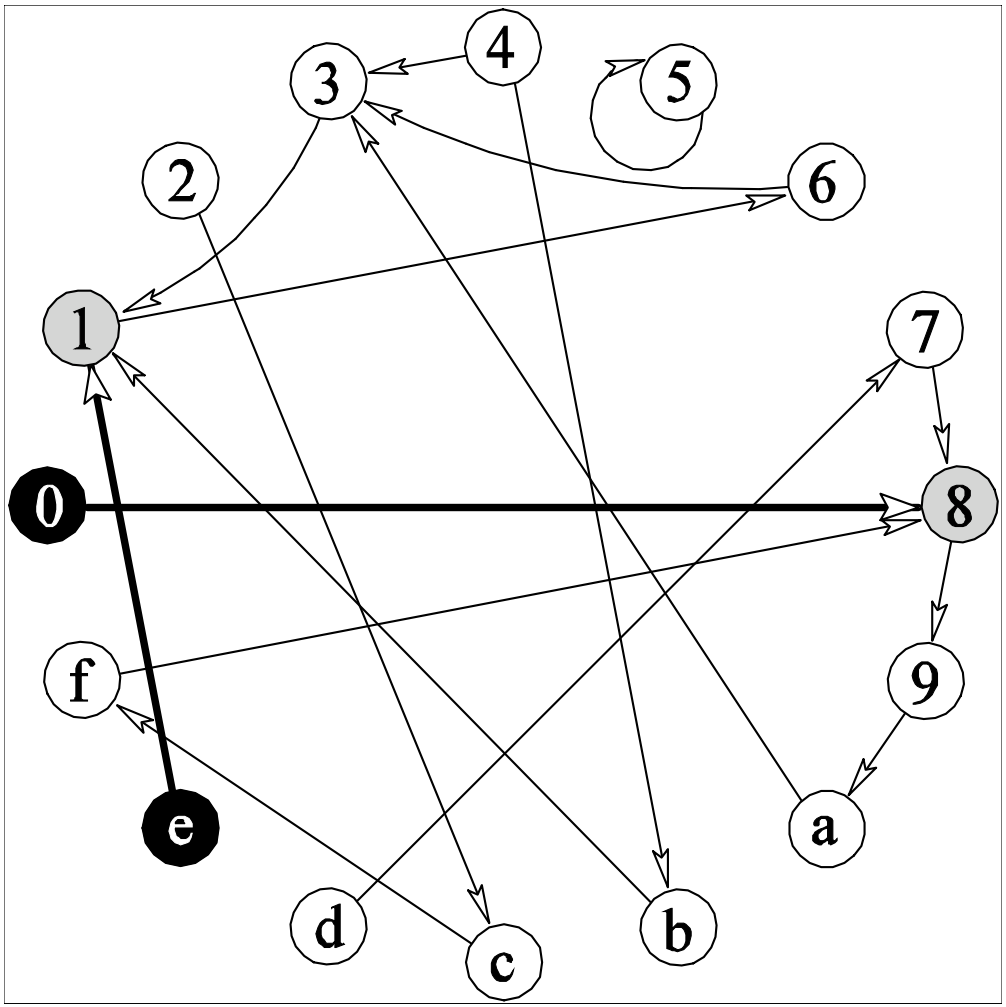
Random Call Model

- **In each round a new contact graph $G_t=(V,E_t)$:**
  - Each node in G_t has out-degree 1
    - chooses random node v out of V
- **Infection models:**
  - Push-Model
    - if u is infected and $(u,v) \in E_t$, then v is infected in the next round
  - Pull-Modell:
    - if v is infected and $(u,v) \in E_t$, then u is infected in the next round

Push Model



Push Model



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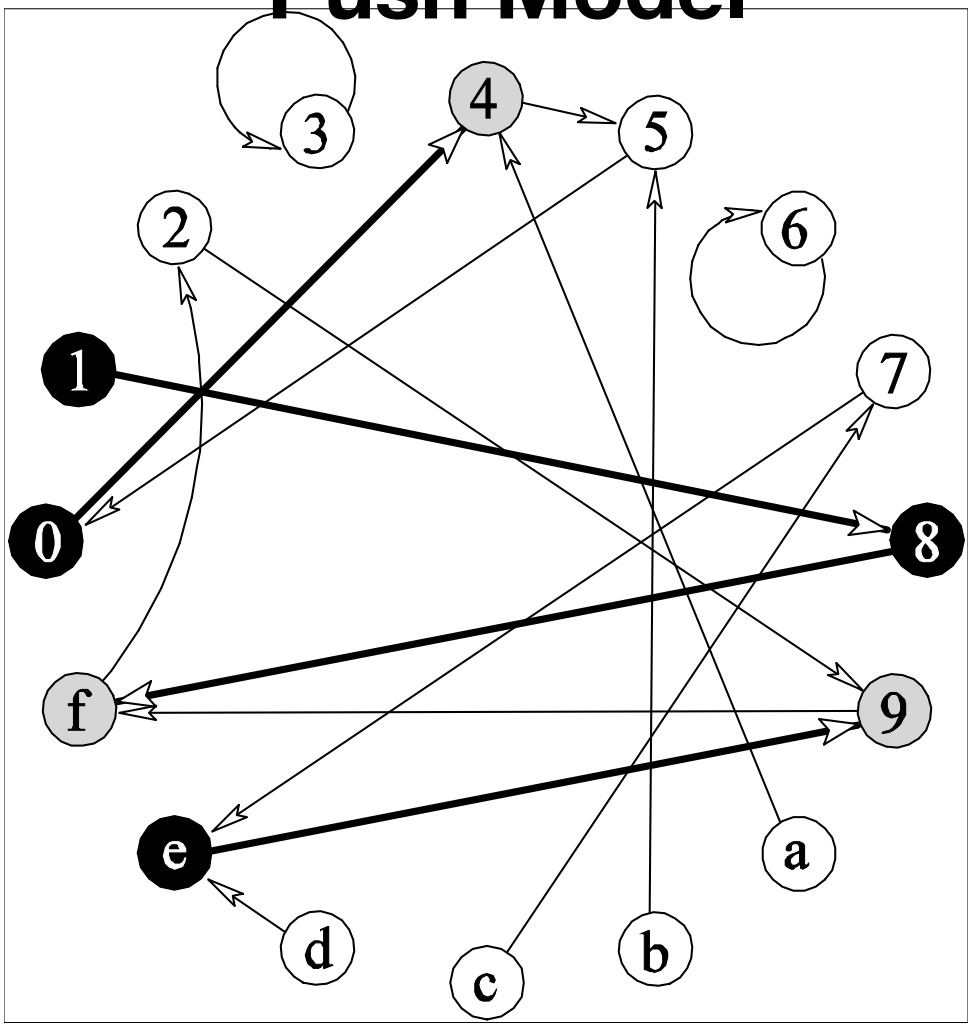

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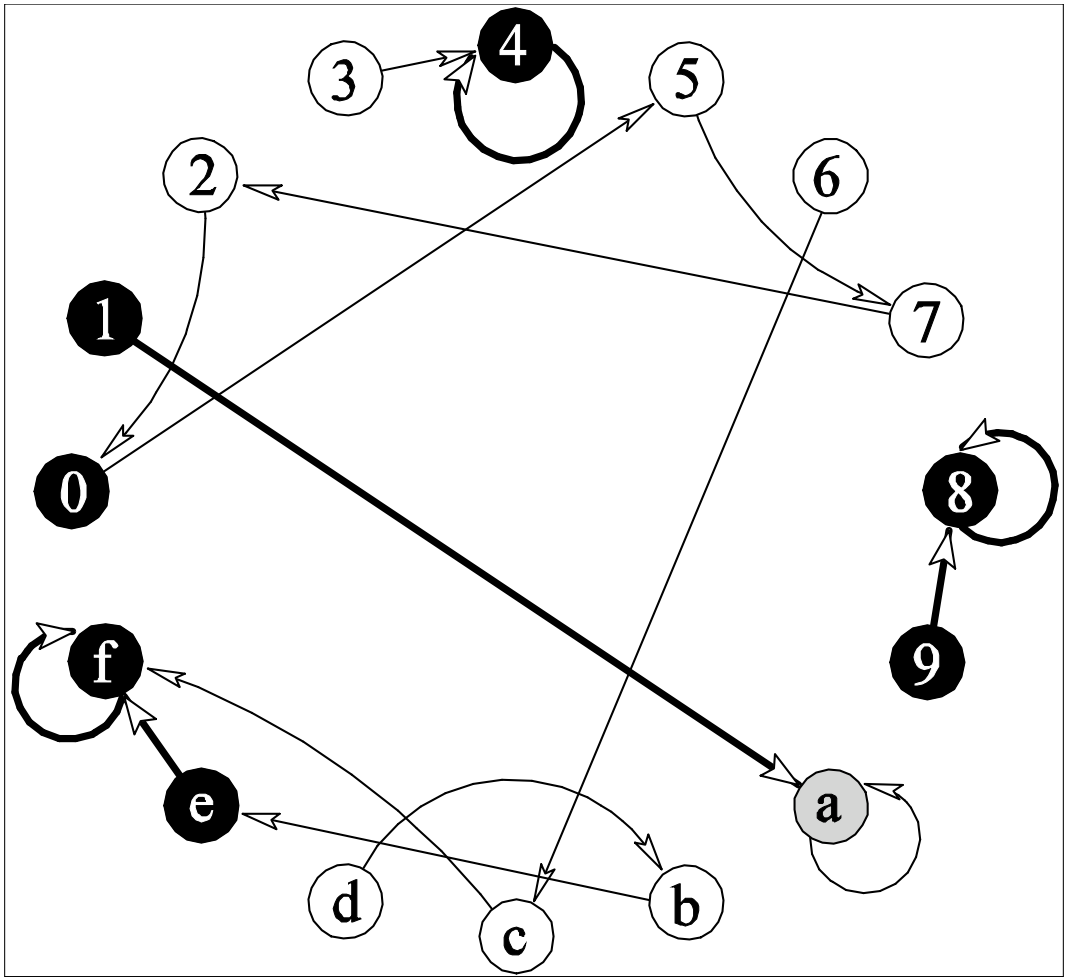

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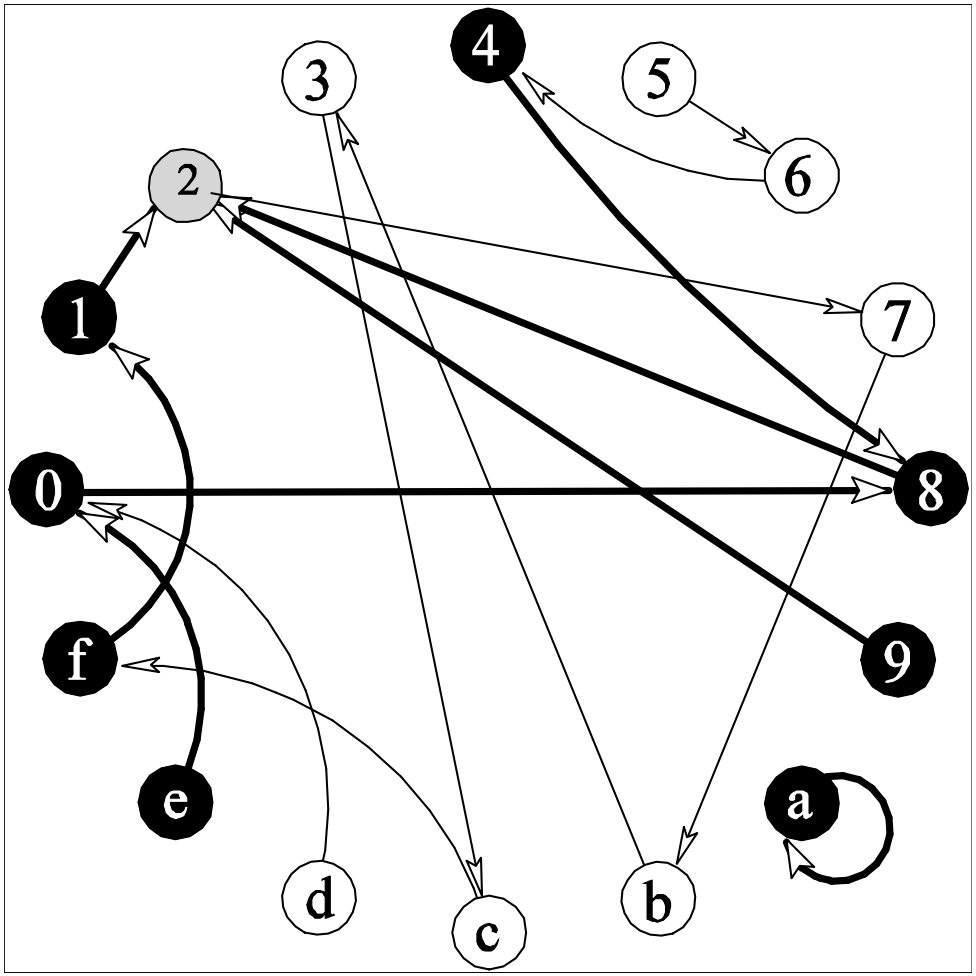

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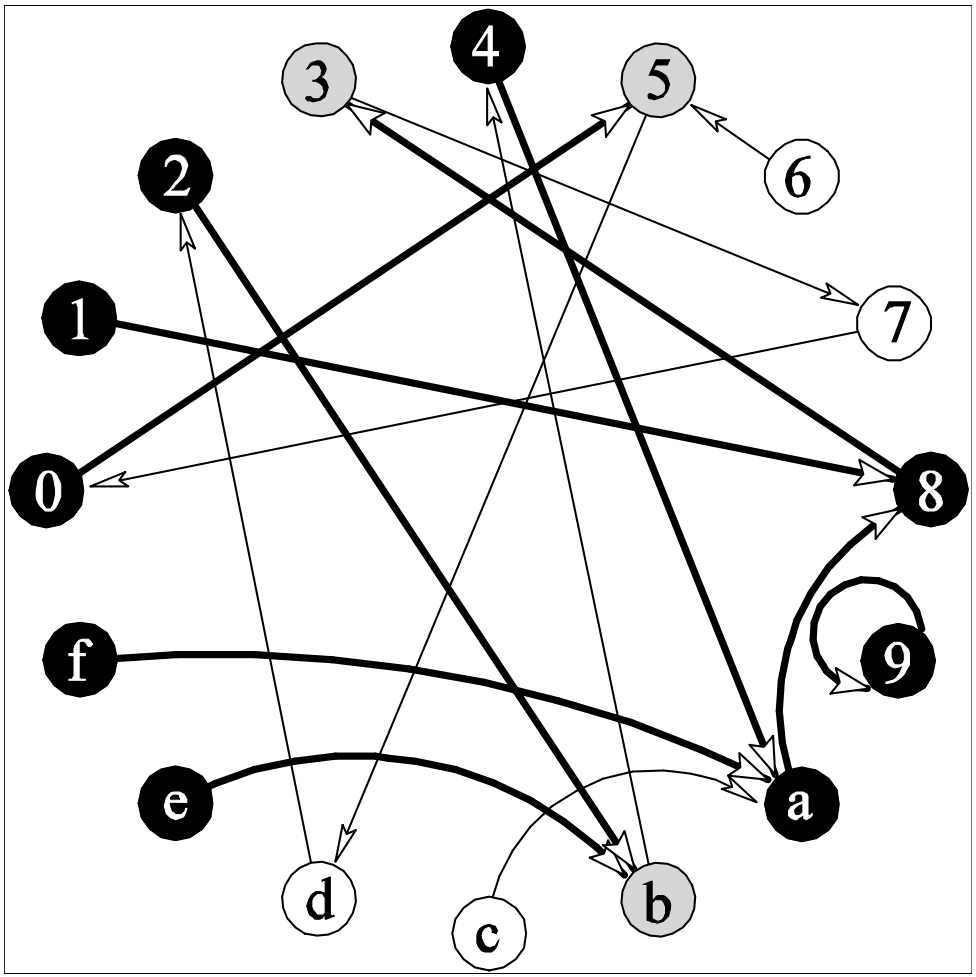

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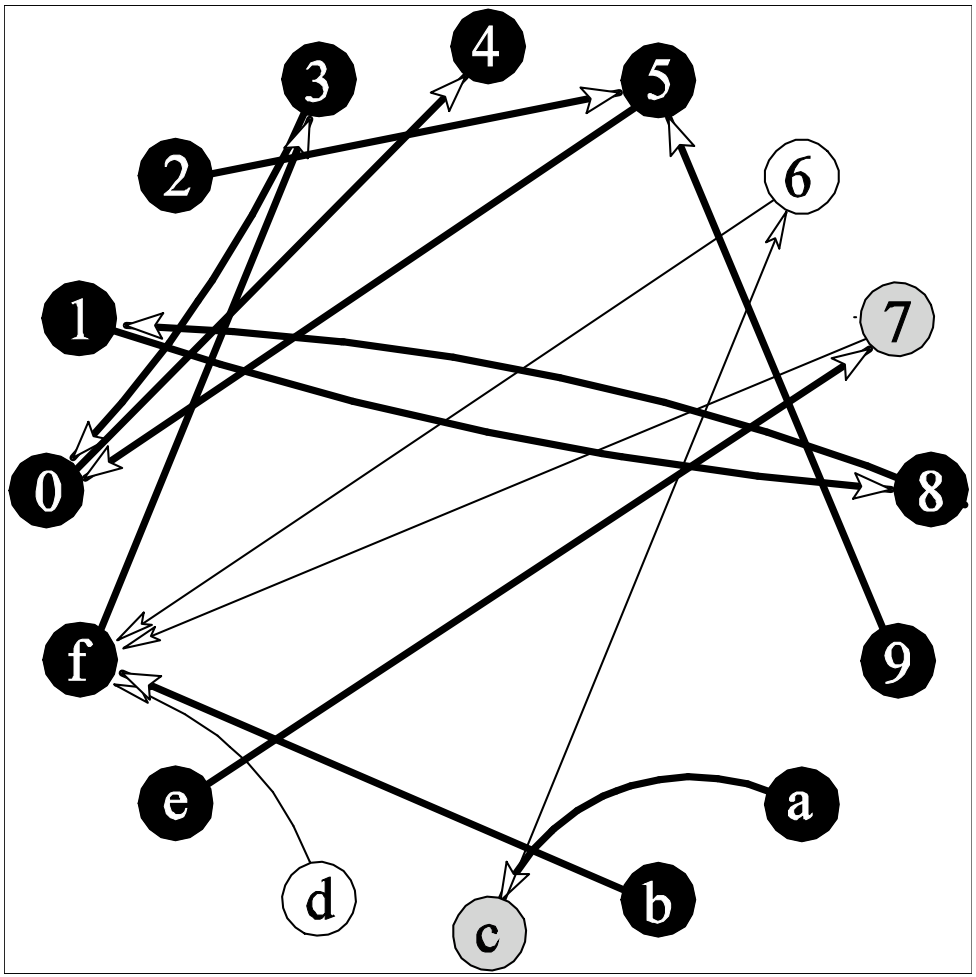

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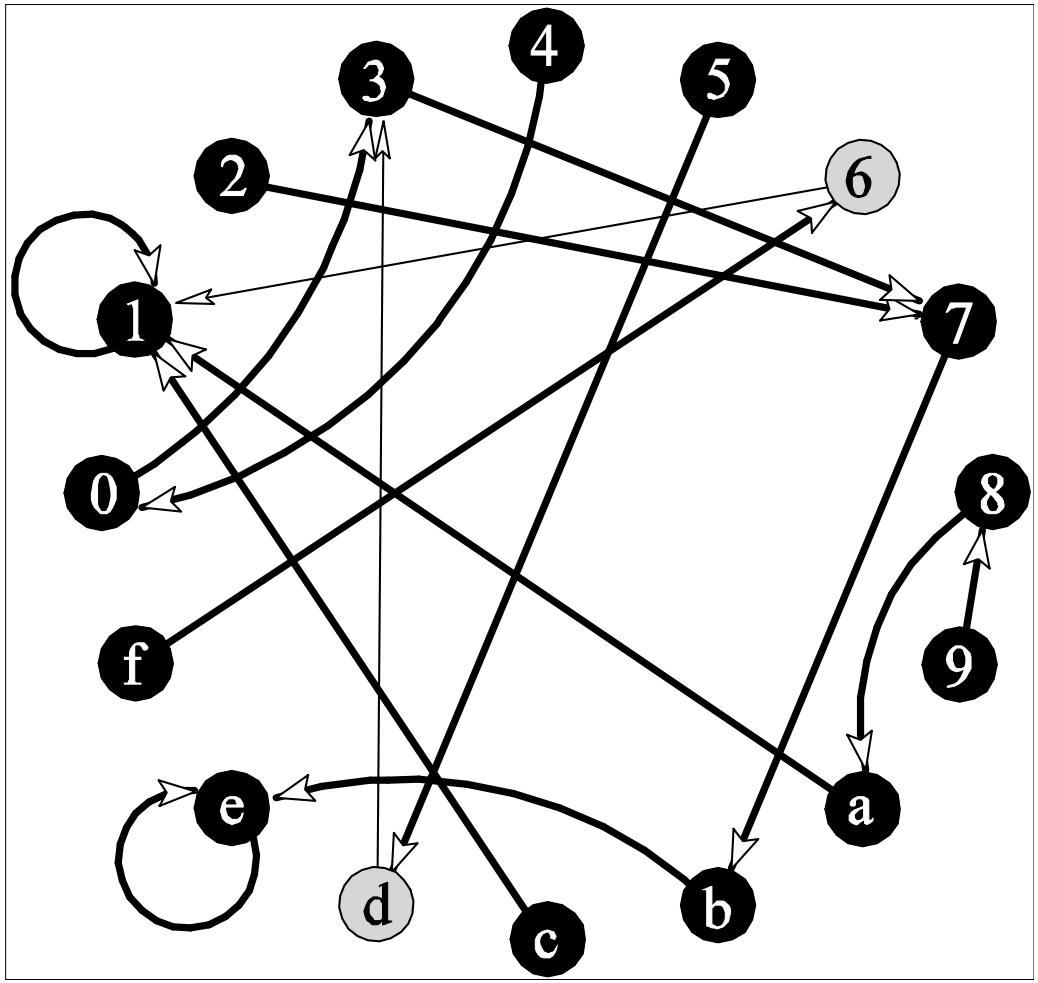


Push Model
Start Phase

- **3 cases for an infected node**
  1. he is the only one infecting a new node
  2. he contacts an already infected node
  3. he infects together with other infected nodes a new node
     - this case is neglected in the prior deterministic case
  - Probability for 1st or 3rd case $s(t) = 1-i(t)$
  - Probability for 2nd case $i(t)$
  - Probability for 3rd case is at most $i(t)$
    - since at most $i(t)$ are infected
- **Probability of infection of a new node, if $i(t) \leq s(t)/2$:**
  - at least $1 - 2i(t)$
- **$E[i(t+1)] \geq i(t) + i(t)(1 - 2\,i(t)) = 2i(t) - 2i(t)^2 \approx 2\,i(t)$**

Push Model
Start phase & Exponential Growth

- **If i(t) ≤ s(t)/2:**
  - E[i(t+1)] ≥ 2 i(t) – 2i(t)2 ≈ 2 i(t)
- **Start phase: I(t) ≤ 2 c (ln n)²**
  - Variance of i(t+1) relatively large
  - Exponential growth starts after some O(1) with high probability
- **Exponential growth: I(t) ∈ [2 c (ln n)², n/(log n)]**
  - Nearly doubling of infecting nodes with high probability, i.e. 1-O(n^{-c})

- **Proof by Chernoff-Bounds**
  - For independent random variables $X_i \in \{0,1\}$ with $X_m = \sum_{i=1}^{m} X_i$
  - and any $0 \leq \delta \leq 1$

$$\mathrm{P}[X_m \leq (1-\delta)\mathbf{E}[X_m]] \leq e^{-\delta^2 \mathbf{E}[X_m]/2}$$

  - Let $\delta = 1/(\ln n)$
  - $\mathbf{E}[X_m] \geq 2\,c\,(\ln n)^3$
  - Then $\delta^2\,\mathbf{E}[X_m]/2 \geq c \ln n$
  - This implies

$$\mathrm{P}[X_m \leq (1-\delta)\mathbf{E}[X_m]] \leq e^{-\delta^2 \mathbf{E}[X_m]/2} \leq n^{-c}$$

Chernoff Bounds

- **Bernoulli-experiment**
  - result 1 with probability p
  - result 0 with probability 1-p
- **Theorem Chernoff-Hoeffding**
  - Let $x_1,...,x_n$ independent Bernoulli-experiments with
    - $P[x_i=1]=p$
    - $P[x_i=0]=1-p$
    - let $$S_n = \sum_{i=1}^{n} x_i$$
  - Then for all c>0

$$\mathbf{P}\left[S_n \geq (1+c)\mathbf{E}[S_n]\right] \leq e^{-\frac{1}{3}\min\{c,c^2\}pn}$$

  - For all $c \in [0,1]$

$$\mathbf{P}\left[S_n \leq (1-c)\mathbf{E}[S_n]\right] \leq e^{-\frac{1}{2}c^2pn}$$

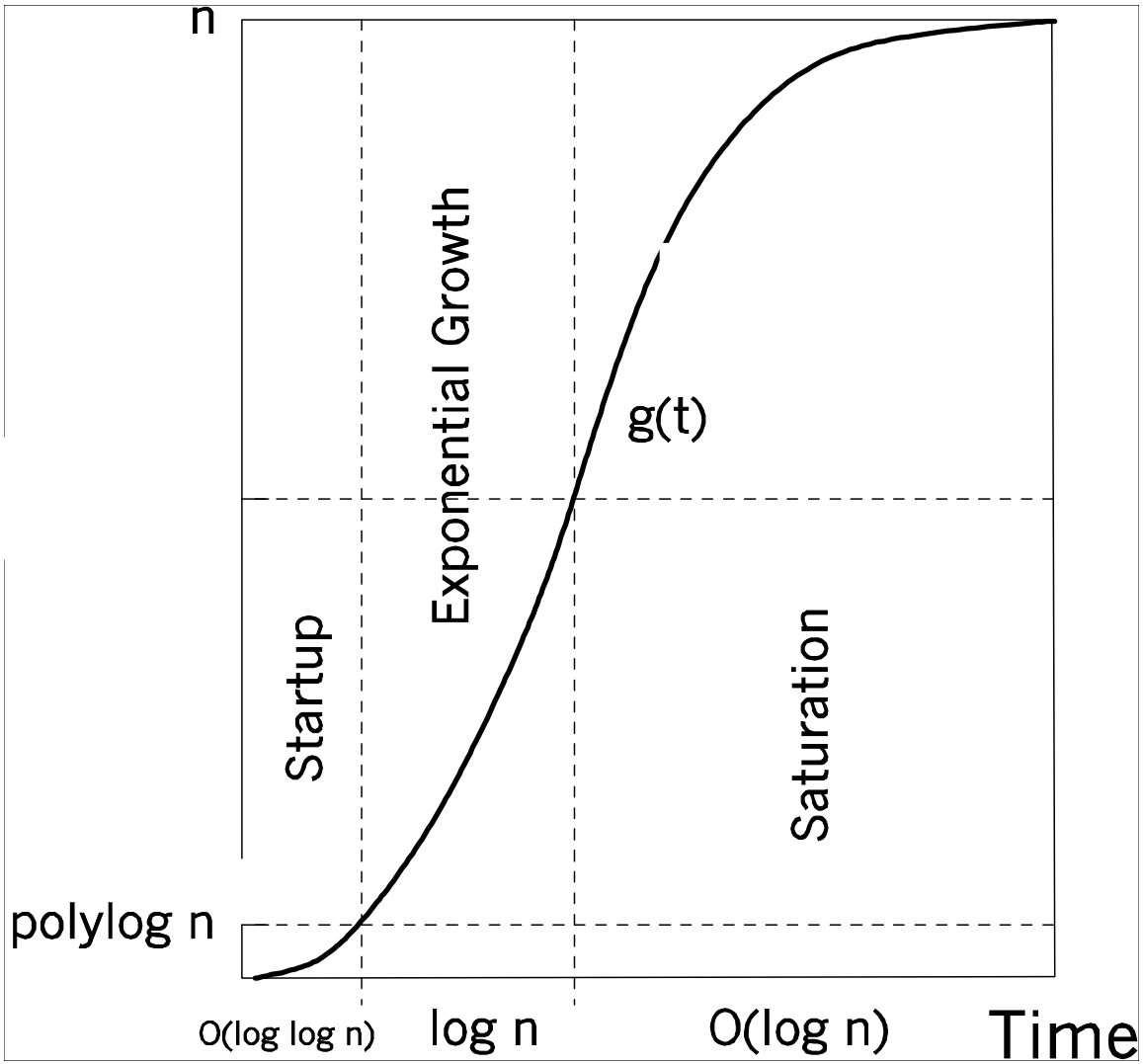
Push Model
Middle Phase & Saturation

- Probability of infections of a new node if i(t) ≤ s(t)/2**: 1 – 2i(t)**
  - **E[i(t+1)] ≥ 2 i(t) – 2i(t)² ≈ 2 i(t)**
- **Middle phase I(t) ∈ [n/(log n), n/3]**
  - term **2i(t)² ≥ 2i(t)/(log n)** cannot be neglected anymore
  - Yet, **2i(t) – 2i(t)² ≥ 4/3 i(t)** still implies expontential growth, but with base < 2
- **Saturation: I(t) ≥ n/3**
  - Probability that a susceptible node is not contacted by I(t) = c n infected nodes:

$$\left(1-\frac{1}{n}\right)^{cn} = \left(\left(1-\frac{1}{n}\right)^{n}\right)^{c} \leq \frac{1}{e^{c}}$$

- This implies a constant probability for infection ≥ **$1 - e^{-1/3}$** und ≤ **$1 - e^{-1}$**
- Hence **$E[s(t+1)] \leq e^{-i(t)}\, s(t) \leq e^{-1/3}\, s(t)$**
- Chernoff-bounds imply that this holds with high probability
- Exponential shrinking of susceptible nodes
- Base converges to 1/e

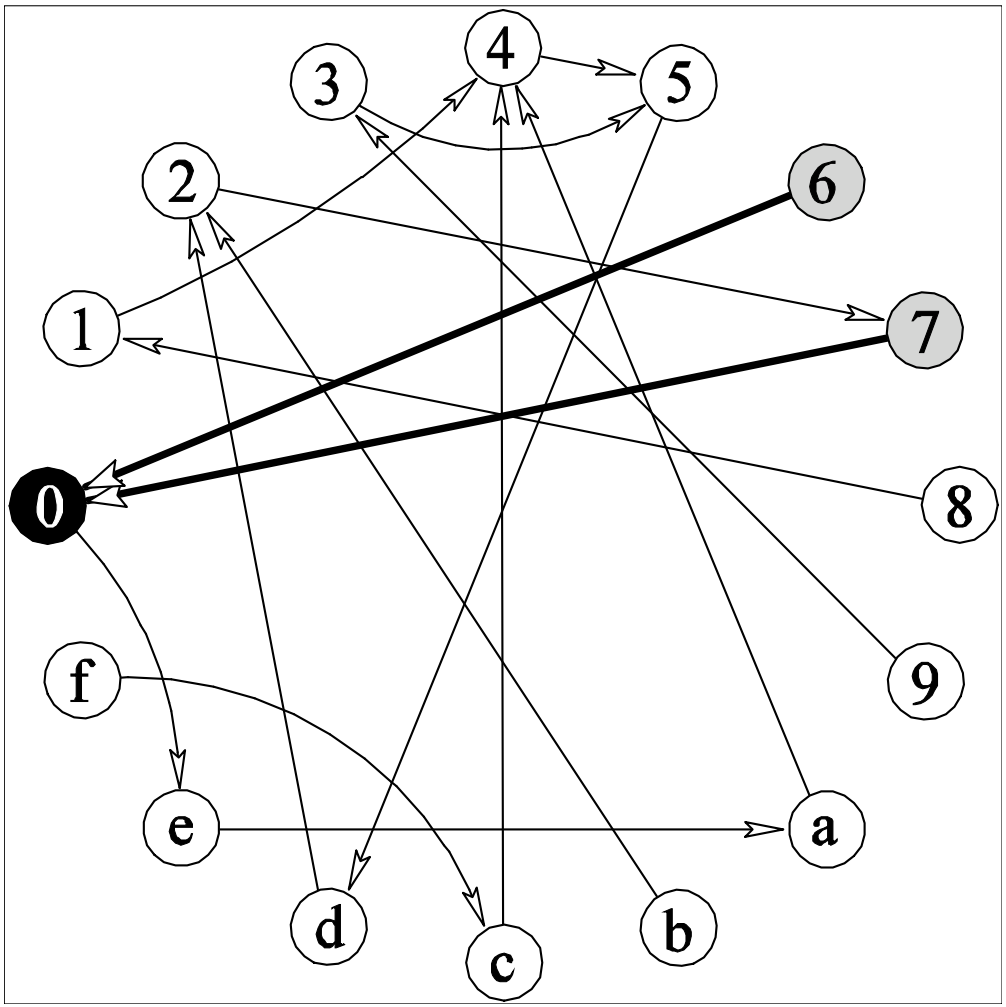
Push Model


n
Exponential Growth
g(t)
Startup
Saturation
polylog n
O(log log n)
log n
O(log n)
Time

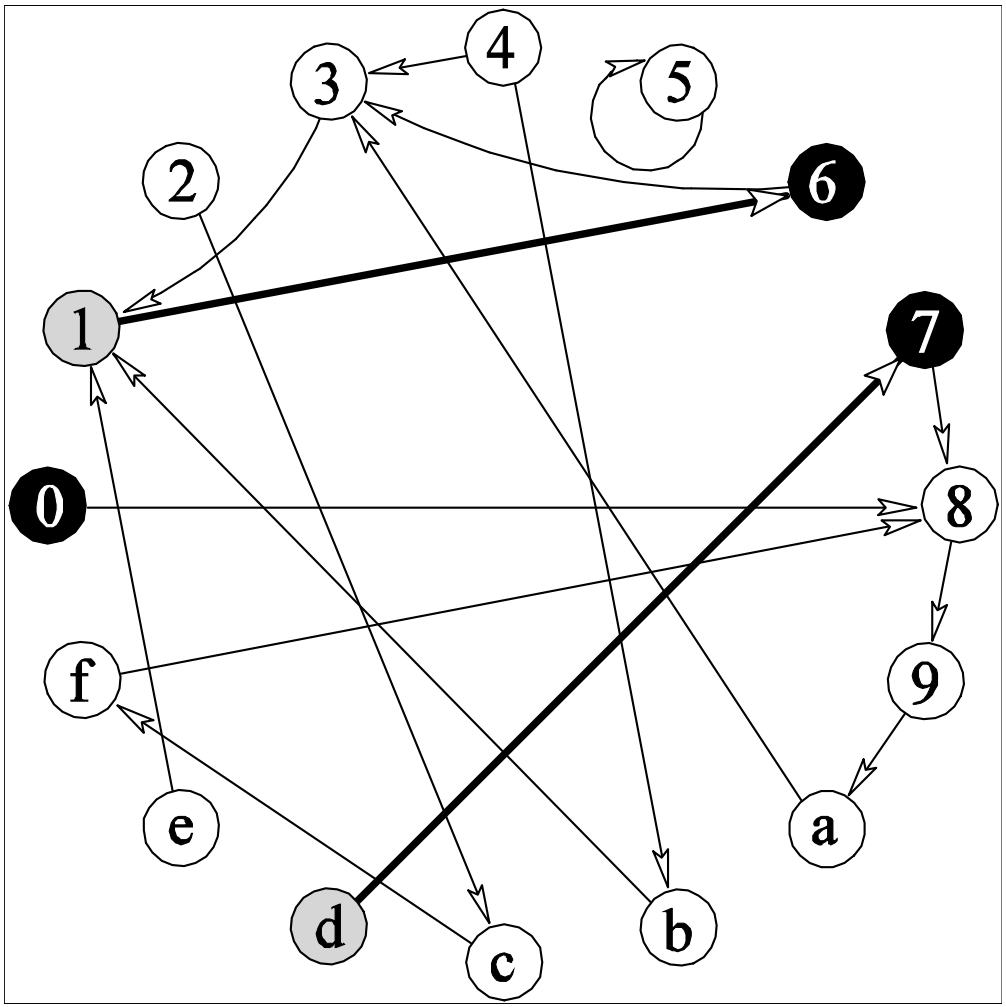
Random Call Model

- **Infection models:**
  - Push Model
    - if u is infected and $(u,v) \in E_t$, then v is infected in the next round
  - Pull Model
    - if v is infected and $(u,v) \in E_t$, then u is infected in the next round

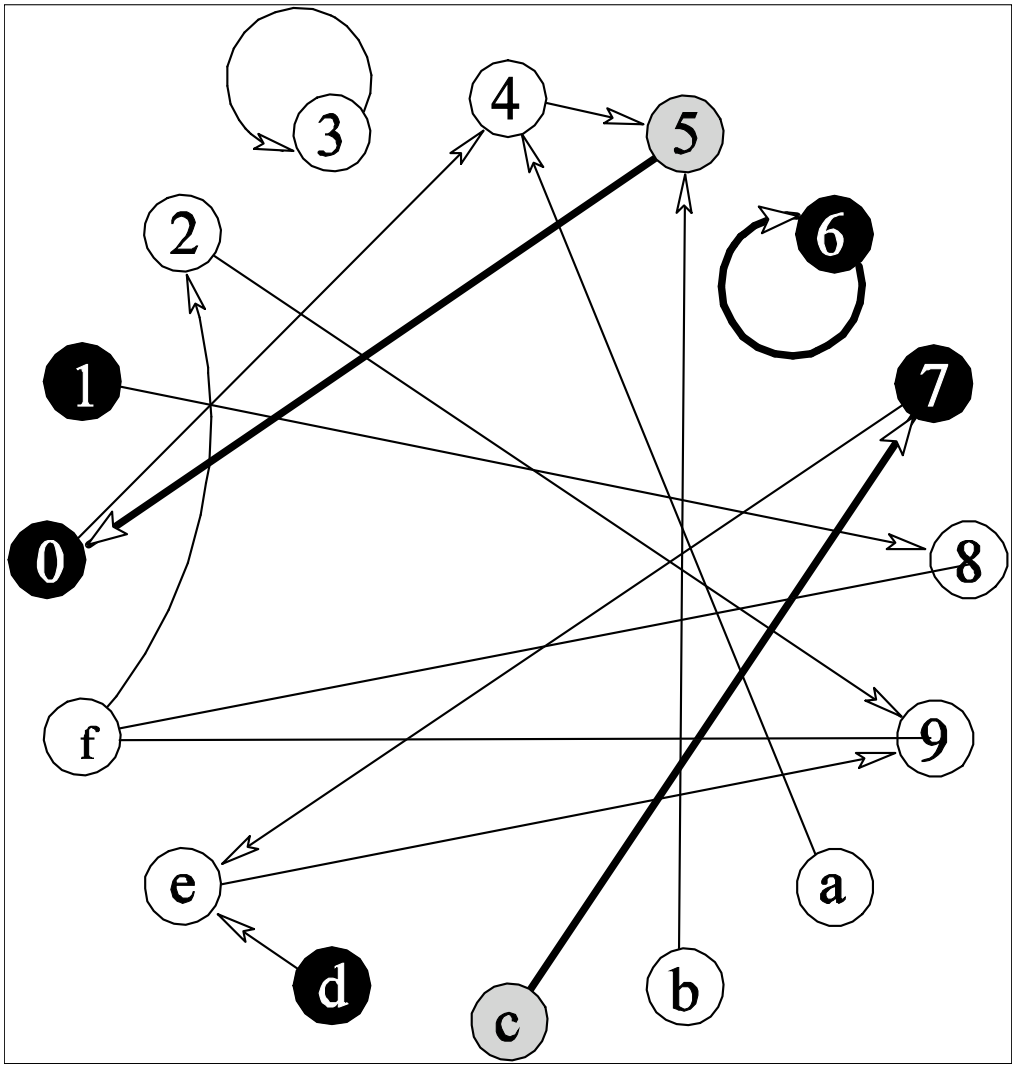
Pull Model



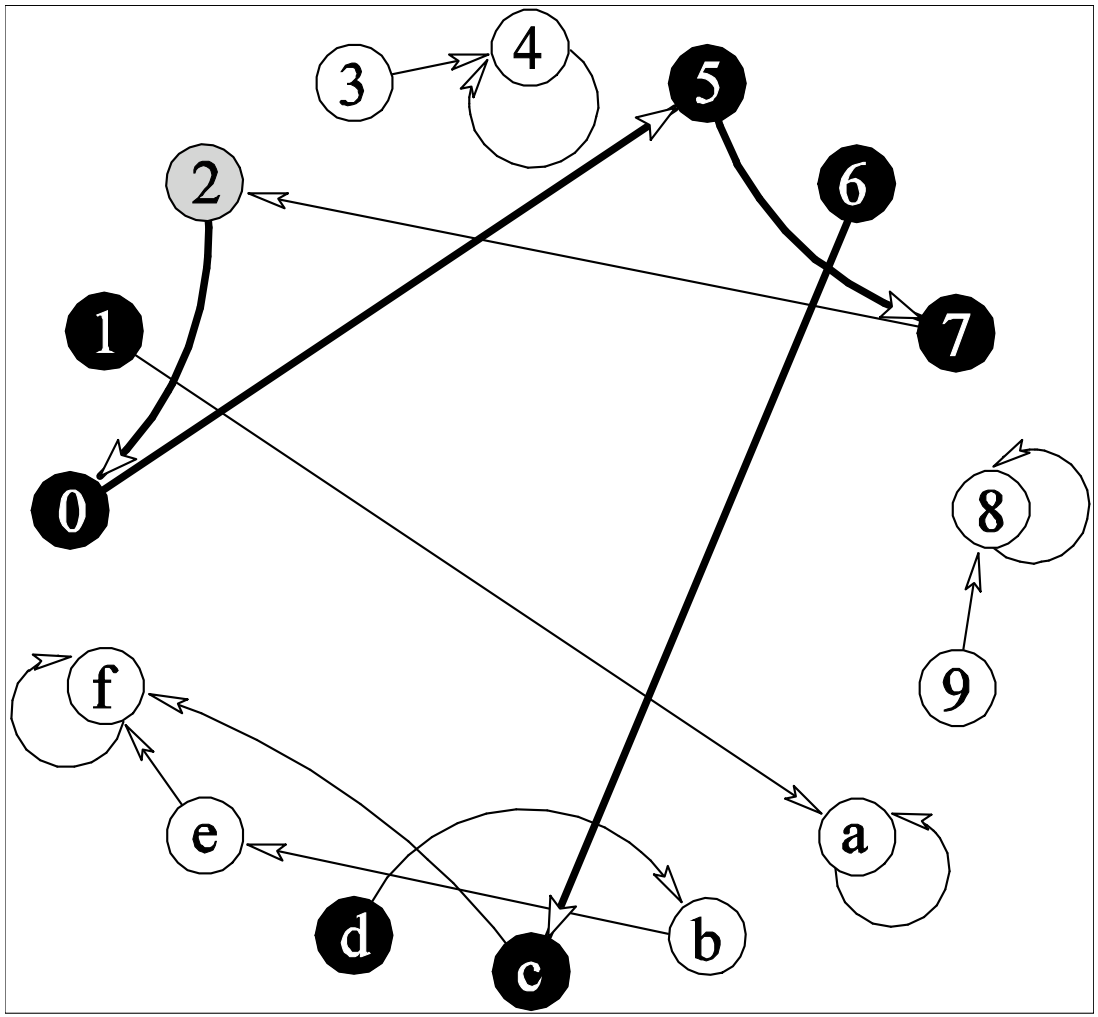
Pull Model



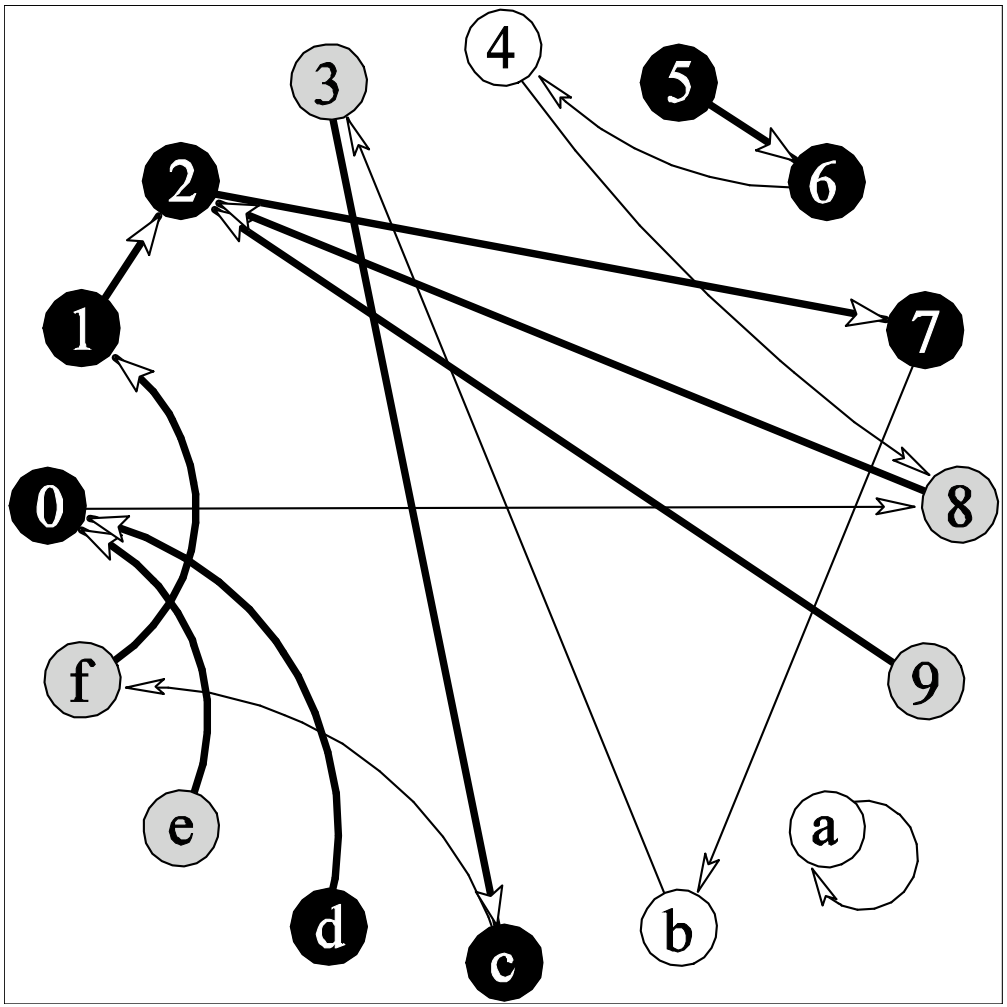
Pull Model



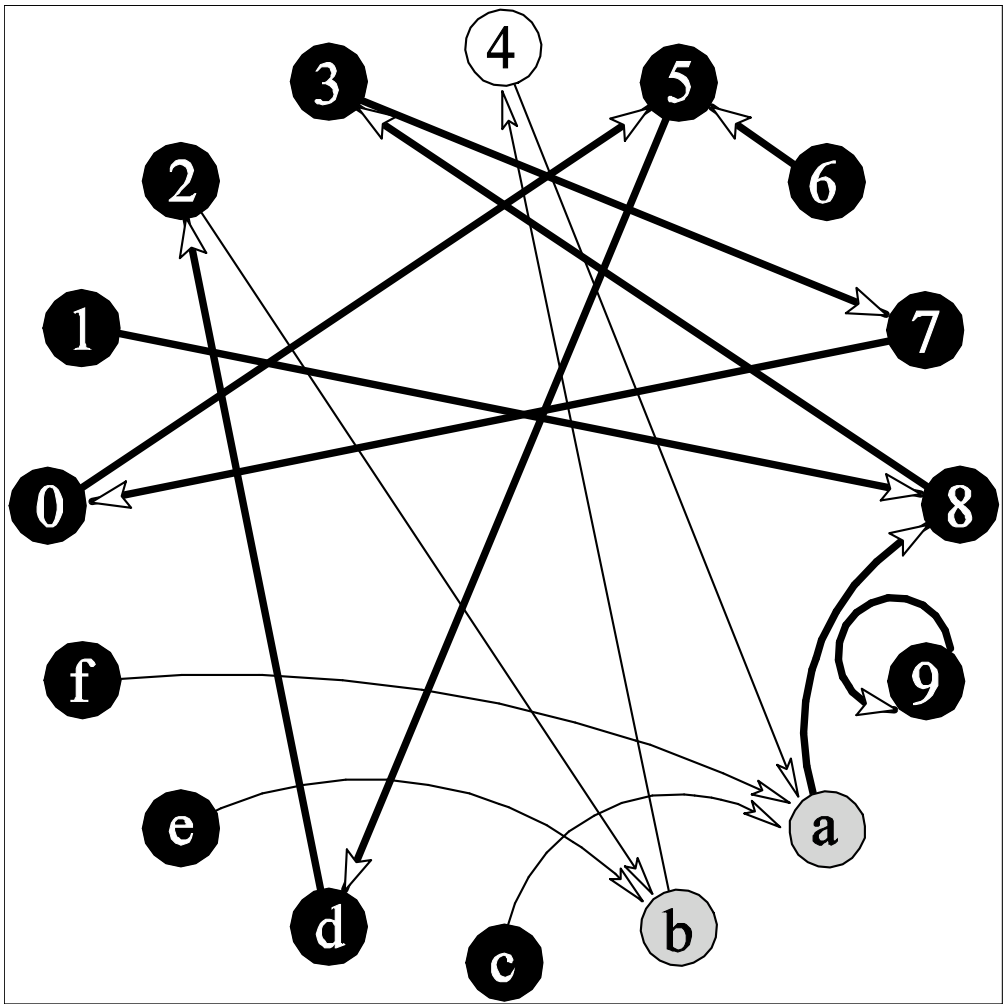
Pull Model



Pull Model



Pull Model

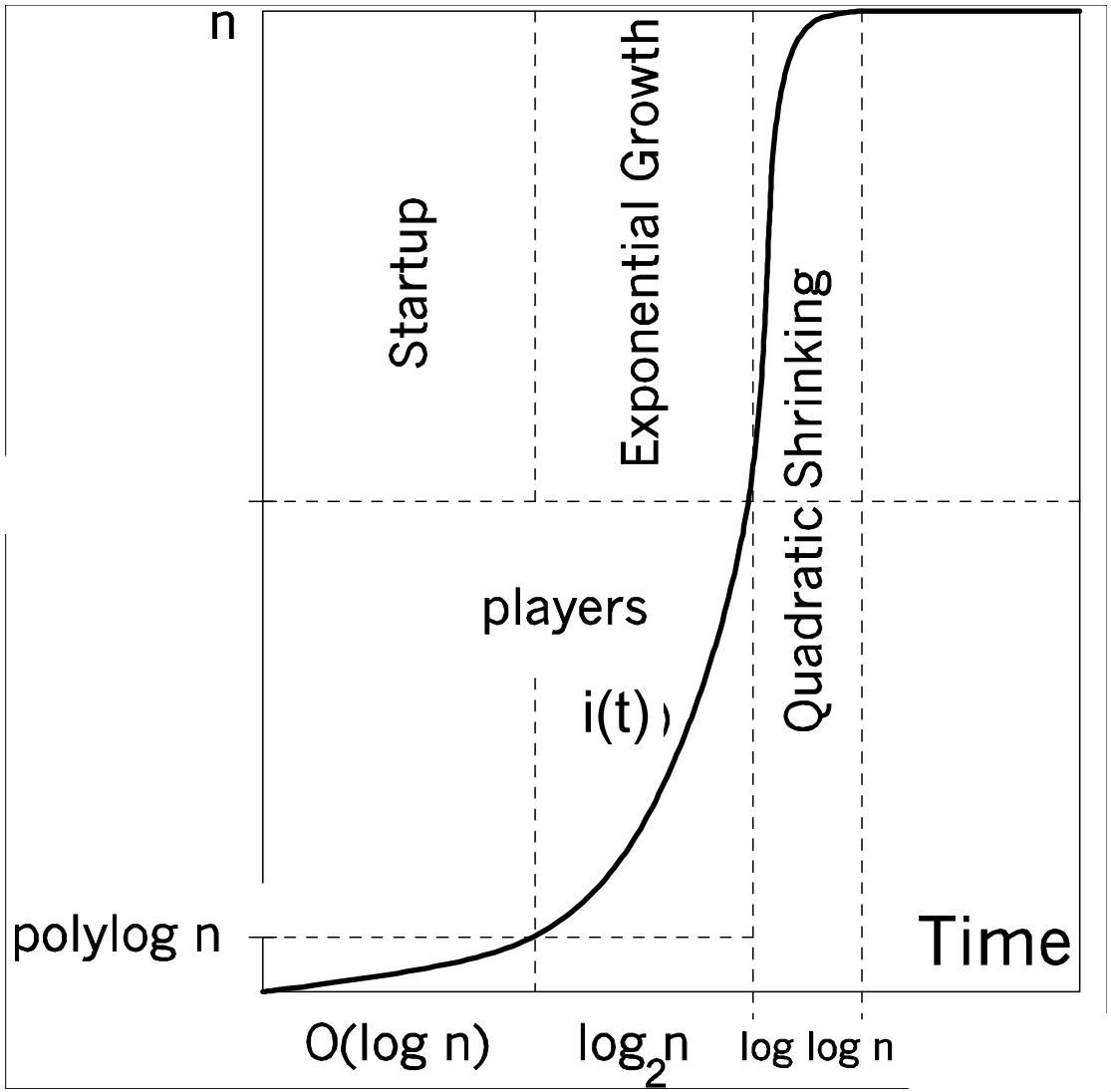


Pull Model

- **Consider**
  - an susceptible node and I(t) infected nodes
- **Probability that a susceptible node contacts an infected node: i(t)**
  - E[s(t+1)]
    = s(t) – s(t) i(t)
    = s(t) (1 – i(t)) = $s(t)^2$
  - E[i(t+1)]
    = 1-$s(t)^2$
    = 1 – $(1 – i(t))^2$
    = 2 i(t) – $i(t)^2$ ≈ 2 i(t) for small i(t)

- **Problem**
  - if i(t) ≤ $(\log n)^2$ then exponential growth is not with high probability
  - O(log n) steps are needed to start eh growth with high probability
    - yet in the expectation it grows exponentially
- **After this phase**
  - If s(t) ≤ ½
    - then the share of susceptible nodes is squared in each step
  - This implies E[s(t+ O(log log n))] = 0,
  - If i(t) ≥ ½ then after O(log log n) steps all nodes are infected with high probability

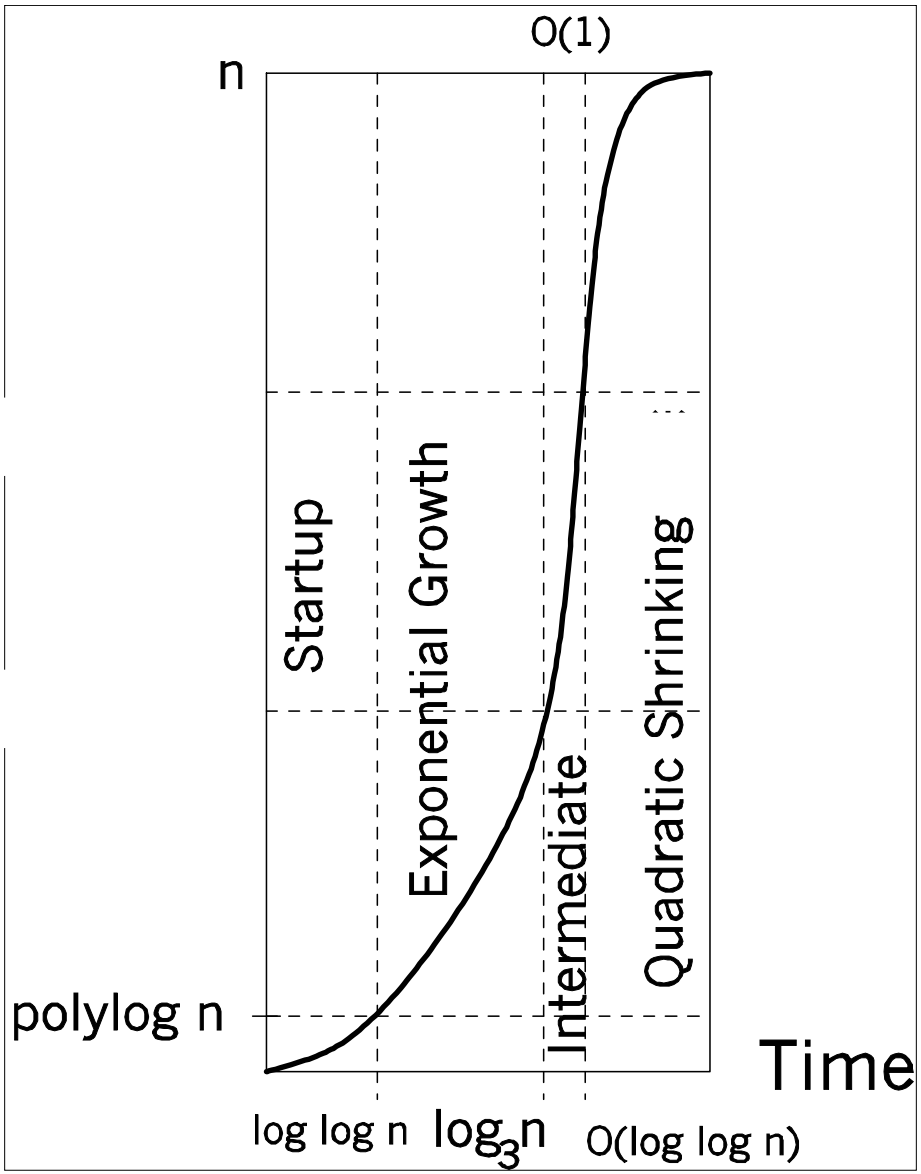
Pull Model


n
Startup
Exponential Growth
Quadratic Shrinking
players
i(t)
polylog n
Time
O(log n)
log2n
log log n

Push&Pull Model

- **Combines growth of Push and Pull**
- **Start phase: $i(t) \leq 2\,c\,(\ln n)^2$**
  - Push causes doubling of i(t) after O(1) rounds with high probability
- **Exponential growth: $I(t) \in [2\,c\,(\ln n)2, n/(\log n)]$**
  - Push and Pull nearly triple in each round with high probability:
    - $i(t+1) \geq 3\,(1\text{-}1/(\log n))\,i(t)$
- **Middle phase: $I(t) \in [n/(\log n), n/3]$**
  - Push and Pull
    - slower exponential growth
- **Quadratic shrinking: $I(t) \geq n/3$**
  - caused by Pull:
  - $E[s(t+1)] \leq s(t)^2$
  - The Chernoff bound implies with high probability
  - $s(t+1) \leq 2\,s(t)^2$
  - so after two rounds for $s(t) \leq 1/2^{1/2}$
    - $s(t+2) \leq s(t)^2$ w.h.p.

Push&Pull Model



Max-Counter Algorithm

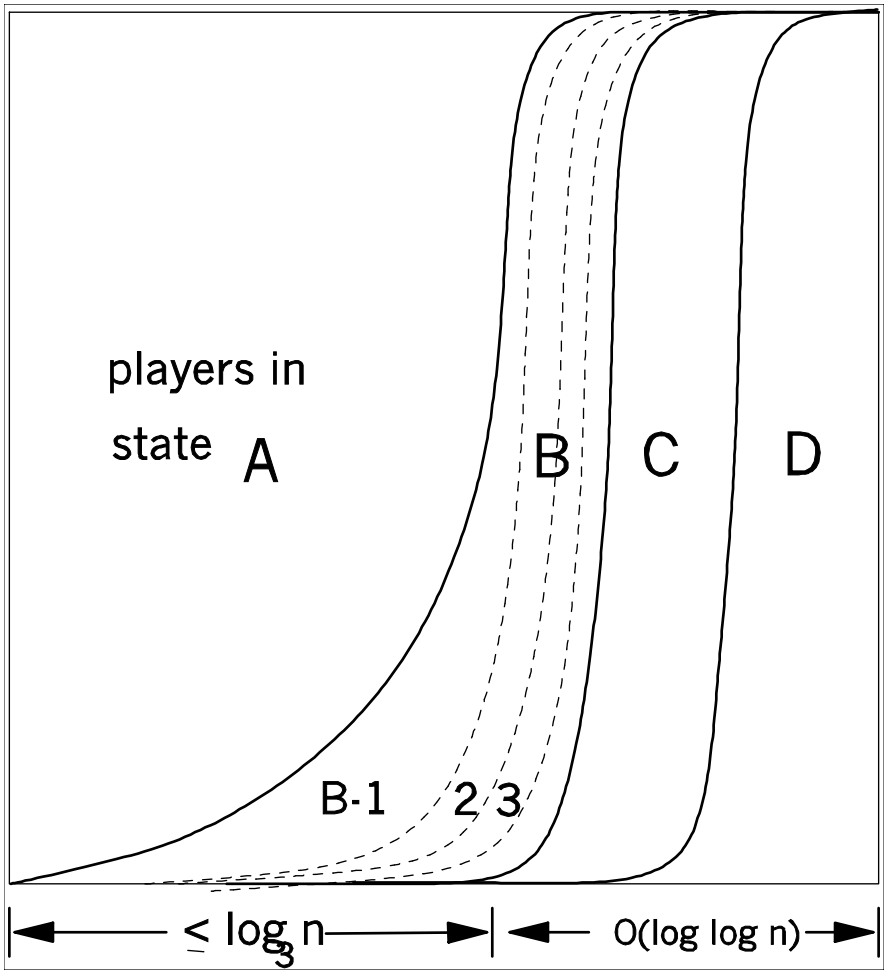
- **Simple termination strategy**
  - If the rumor is older than max_{ctr}, then stop transmission
- **Advantages**
  - simple
- **Disadvantage**
  - Choice of max_{ctr} is critical
  - If max_{ctr} is too small then not all nodes are informed
  - If max_{ctr} is too large, then the message overhead is $\Omega(n\ max_{ctr})$
- **Optimal choice for push-communication**
  - $max_{ctr} = O(\log n)$
  - Number of messages: $O(n \log n)$
- **Pull communication**
  - $max_{ctr} = O(\log n)$
  - Number of messages: $O(n \log n)$
- **Push&Pull communication**
  - $max_{ctr} = \log_3 n + O(\log \log n)$
  - Number of messages: $O(n \log \log n)$

Shenkers Min-Counter Algorithm

- **Only is the rumor is seen as old then contact partners increase the age-counter**
- **Shenkers Min-Counter-Algorithmus für max_{ctr} = O(log log n)**
  - Every player P stores age-variable $ctr_R(P)$ for each rumor R
  - A: player P does not know the rumor:
    - $ctr_R(P) \leftarrow \mathbf{1}$
  - B: If player P sees rumor for the first time
    - $ctr_R(P) \leftarrow 1$
  - B: If partners $Q_1, Q_2, \ldots, Q_m$ communicate with P in a round
  - If $min_i\{ctr_R(Q_i)\} \geq ctr_R(P)$ then
    - $ctr_R(P) \leftarrow ctr_R(P) + 1$
  - C: If $ctr_R(P) \geq max_{ctr}$ then
    - tell the rumor for max_{ctr} more rounds
    - then D: stop sending the rumor
- **Theorem**
  - Shenkers Min-Counter algorithms informs all nodes using Push&Pull-communication in $\log_3 n + O(\log \log n)$ rounds with probability $1-n^{-c}$, using at most O(n log log n) messages.

Shenker‘s Min-Counter-Algorithm

- **Theorem**
  - Shenkers Min-Counter algorithms informs all nodes using Push&Pull-communication in $\log_3 n + O(\log \log n)$ rounds with probability $1-n^{-c}$, using at most $O(n \log \log n)$ messages.



Peer-to-Peer Networks

End of 5th Week

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Summer 2008