Christian Schindelhauer Freiburg, 2008-04-23

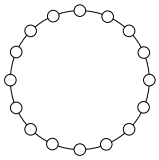
Exercise No. 1

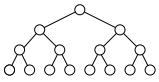
Peer-To-Peer Networks

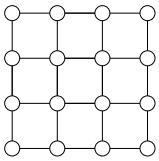
Summer 2008

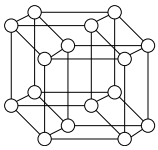
Exercise 1 *Graph Topologies*

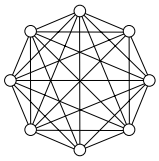
Consider the following graph topologies for n nodes.

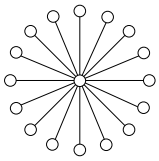
1) ring 

2) balanced binary tree (for $n = 2^k$, $k \in \mathbb{N}$) 

3) two-dimensional torus (for $n = k^2$, $k \in \mathbb{N}$) 

4) hypercube (for $n = 2^k$, $k \in \mathbb{N}$) 

5) complete graph 

6) star 

Indicate for each of the graph topologies the following parameters and sort them according to their asymptotical order.

a) maximal degree of a node

b) amount of edges

c) diameter of the graph

d) maximal amount of nodes that can be reached in d steps starting in one node

e) minimal amount of nodes that have to be removed such that the graph is no longer connected

Exercise 2 *Hash-Functions*

Given is a hash-table with n slots S_i, $i \in \{0, 1, \ldots, n-1\}$. A hash-function h is a mapping of given integer values to those slots, as uniform as possible. A collision occurs, if the hash-function h maps a value to a slot that is already occupied.

1. Choose a hash-function h_d for $n = 7$ and show how the numbers $4, 13, 25, 34, 46$, and 55 are mapped to the slots S_i.

   - How many collisions occur, how can they be treated?
   - Suppose a slot is removed, or a new slot S_7 is added. How does that change the mapping of h?

2. Suppose each number is mapped to a slot by a uniform random function h_r.

   - Using the same collision treatment as above, how many collisions occur in expectation value?
   - What happens when slots are removed/added?
   - What advantage has h_r compared to h_d if the slots change? Also think about n being much larger than the number of values to store.
   - There is a serious disadvantage of h_r. What is it?