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Exercise No. 2
Peer-To-Peer Networks
Summer 2010

Exercise 1 Consider the following algorithm for a binary tree. Start with one node (root) in the tree. In the beginning of each round choose the root first. Then:

- If the chosen node is an internal node, then choose randomly one of its children.
- If the chosen node is a leaf, then add two new leaves to this node and the round ends.

So, in the first round only the root is present. In the second round the tree has the root and two leaves. In the third round one of the leaves becomes an internal node by attaching two new leaves, and so on.

a) How many nodes does the tree have after n rounds? How many of them are leaves?

b) What is the probability that all internal nodes form a chain in the n-th round?

c) Give the probability that the tree is completely balanced in each of the rounds r_k with $r_k = 2^k$, $k \in \mathbb{N}$.

d) Show that in round n only with polynomial small probability, i.e. n^{-c} for $c > 0$, leaves can occur in depth $O(n \cdot \log n)$.

e) What is the expected asymptotic depth of the tree after n rounds?

Exercise 2 Given $m = c \cdot n$ data. Compute the probability that a given rectangle of size $1/n$ does not receive any data. What is the expected amount of such rectangles?