ALBERT-LUDWIGS-UNIVERSITÄT FREIBURG

Network Protocol Design and Evaluation

07 - Simulation, Part I

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Overview

- **In the last chapters:**
  - Formal Specification, Validation, Design Techniques
  - Implementation → Software Engineering, Lab Courses

- **In this chapter:**
  - Performance evaluation by simulation
  - Simulation models

After the design phase...

- **Implementation and Test?**
  - A good idea, but testing in a real environment requires a lot of effort.
  - You might want to start with a prototype to get some more insights.

- **Alternative evaluation methods?**

Evaluation

- **Methods of performance evaluation:**
  - Experiments: Measuring performance in a concrete example in a real environment.
  - Simulation: Numerical evaluation of a system model in an artificial environment.
  - Analysis: Describing properties of a mathematical abstraction of the system.

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Evaluation Methods

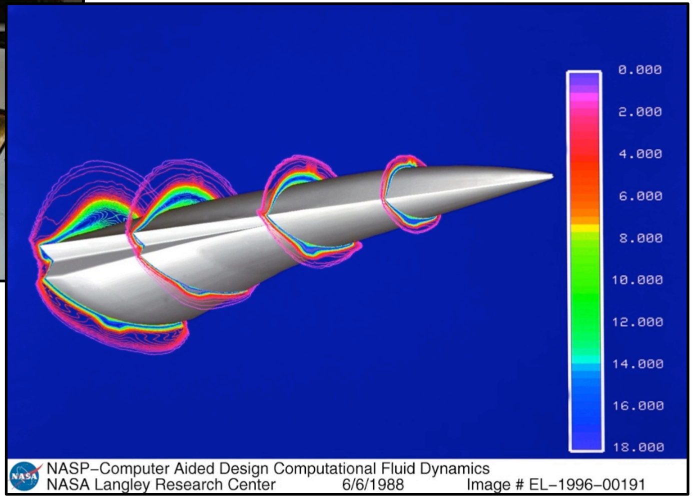
Real World Experiment

Wingtip vortices
Image Credit: NASA

Physical Model

Model in a wind tunnel
Image Credit: NASA

Mathematical Model



Computational Fluid Dynamics
Image Credit: NASA

Mathematical System Models

- **Static vs. dynamic**
  - Static models cover a certain fixed state of a system. State changes are not considered
  - Dynamic models reflect the system's state changes over time
  - The time model can be continuous or discrete

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Continuous vs. Discrete Models

- **Time-continuous models**
  - State variables are continuous functions over time, e.g. description of variable changes by differential equations.
- **Time-discrete models**
  - State changes happen only at discrete time points (state variables do not change in between).
  - We call these time points **events**.

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

System and Models

- The choice of the type of model depends on objectives and feasibility!
- Continuous systems may be described by discrete models and vice versa. Examples:
    - Voice communication (continous system) may be described by a discrete model if digital samples are transmitted.
    - Internet traffic (discrete system with packet transmissions as events) can be described by a continuous model, if the large-scale behaviour is of interest.

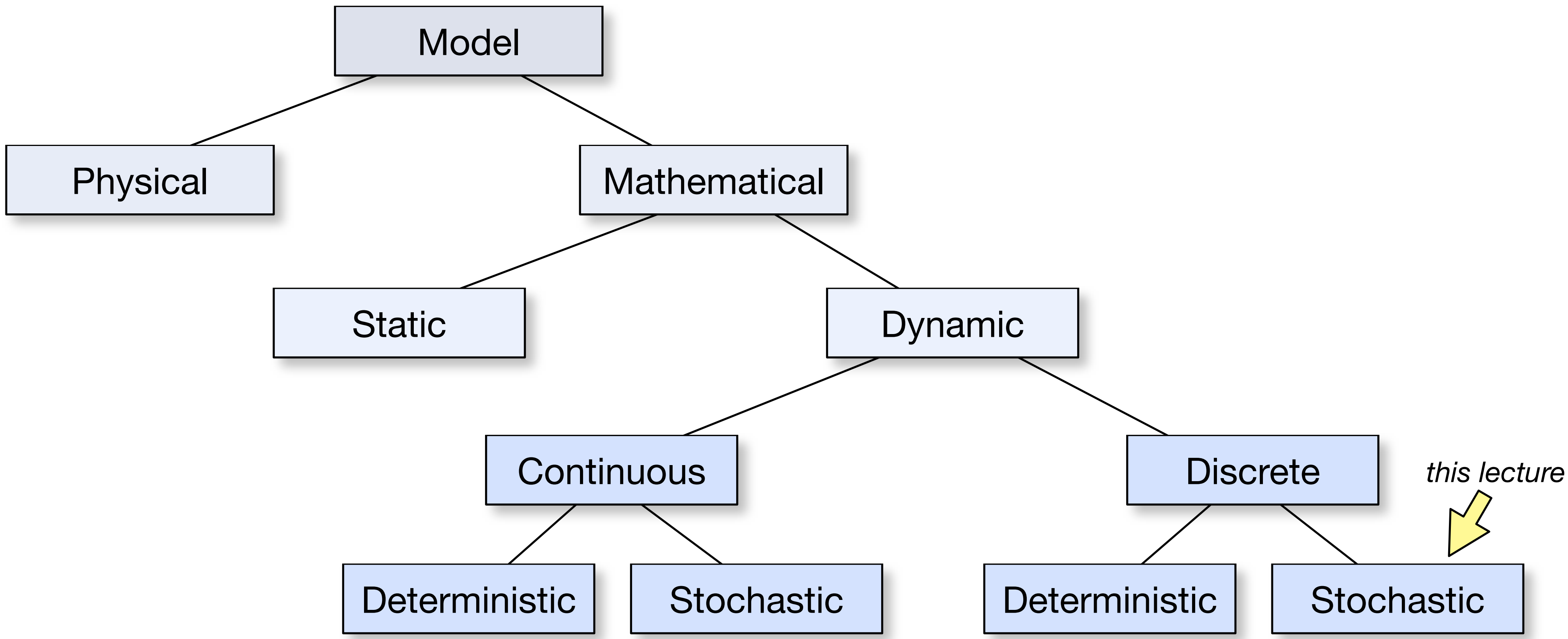
[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Deterministic vs. Stochastic Models

- **Deterministic models**
  - The sequence of state changes depend on the initial state can be completely described
- **Stochastic models**
  - The sequence of state changes depend on random events

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

System Models



[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Evaluation Goals

- The goal of experimental or simulative studies is the evaluation of some system properties
  - Gain insight in system behaviour
  - Get performance estimations
  - Use results to improve the design
  - Reduce cost

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Parameters and Metrics

- Most systems have a set of **parameters** that determine their behaviour
- An evaluation tries to characterize a system by a set of **metrics**.

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Simulation of Discrete Models

- We consider the simulation of communication protocols
- Models are usually dynamic, time-discrete and stochastic

- **Generic procedure:**
  1. Implement the model for system behaviour
  2. Define parameters
  3. Run the simulation
  4. Observe metrics
  5. Evaluate results (this will raise more questions...)

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

A generic simulation model

- ... for discrete event simulation.
- Elements:
  - **Simulation clock**
  - **Event list** (sorted w.r.t. time)

- **Next-event time advance algorithm:**
  - Initialize simulation clock to 0; Initialize future event list.
  - Repeat
    - Advance simulation clock to next event in list
    - Update system state and insert new events in list
  - Until event list empty or simulation time exceeded

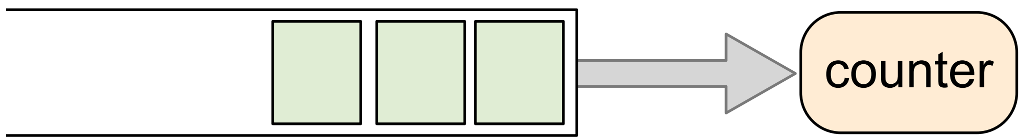
[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Simulation example: Are you being served?

- **Simulating a queue**:
  - Customers wait in a queue to be served.
  - It requires some time to serve each customer.
  - How long do you have to wait?

Modeling a Queue (1)

- **Simulation model:**
  - There is one counter



  - Customers appear at certain time points
  - The service itself requires some time

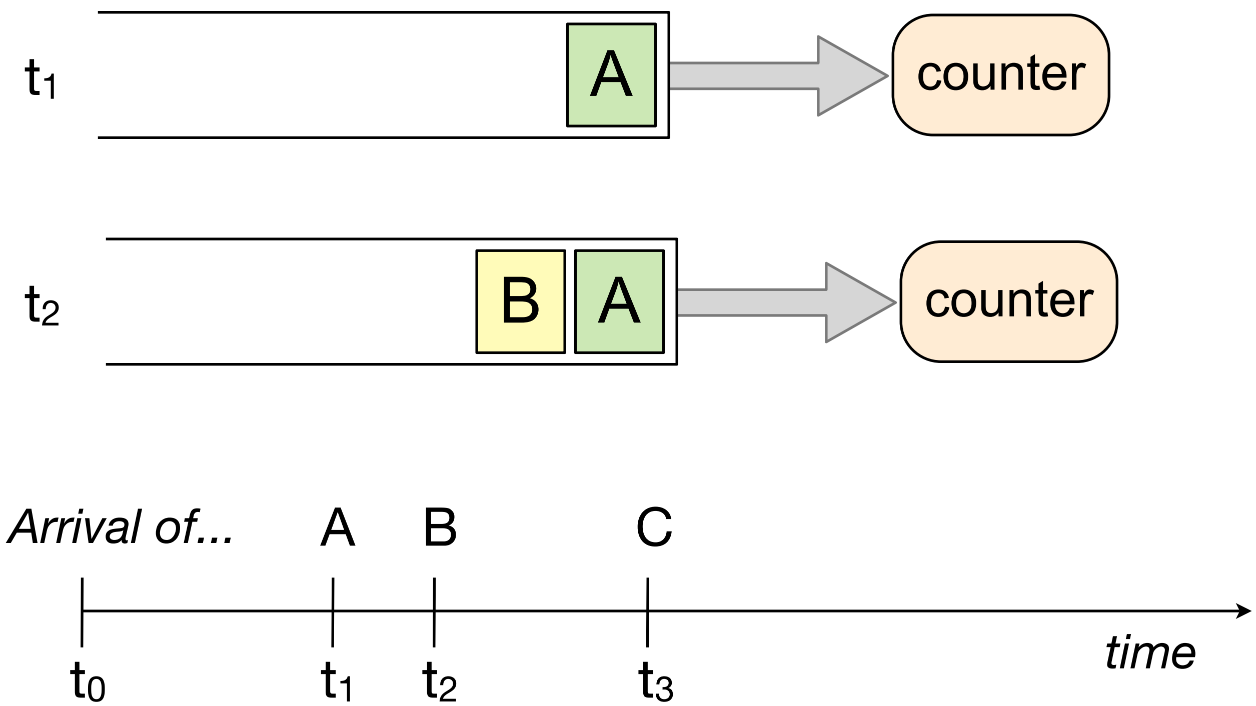
[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Modeling a Queue (2)

- **Parameters:**
  - Patterns of customer arrival and service times:
    - When do customers arrive?
    - How long does it take to serve each customer?
  - Stochastic model: Inter-arrival time as random variable (usually assumed to be exponentially distributed)
- **Metrics:**
  - Waiting time (average, maximum)
  - Queue length (average, maximum)
  - Server utilization

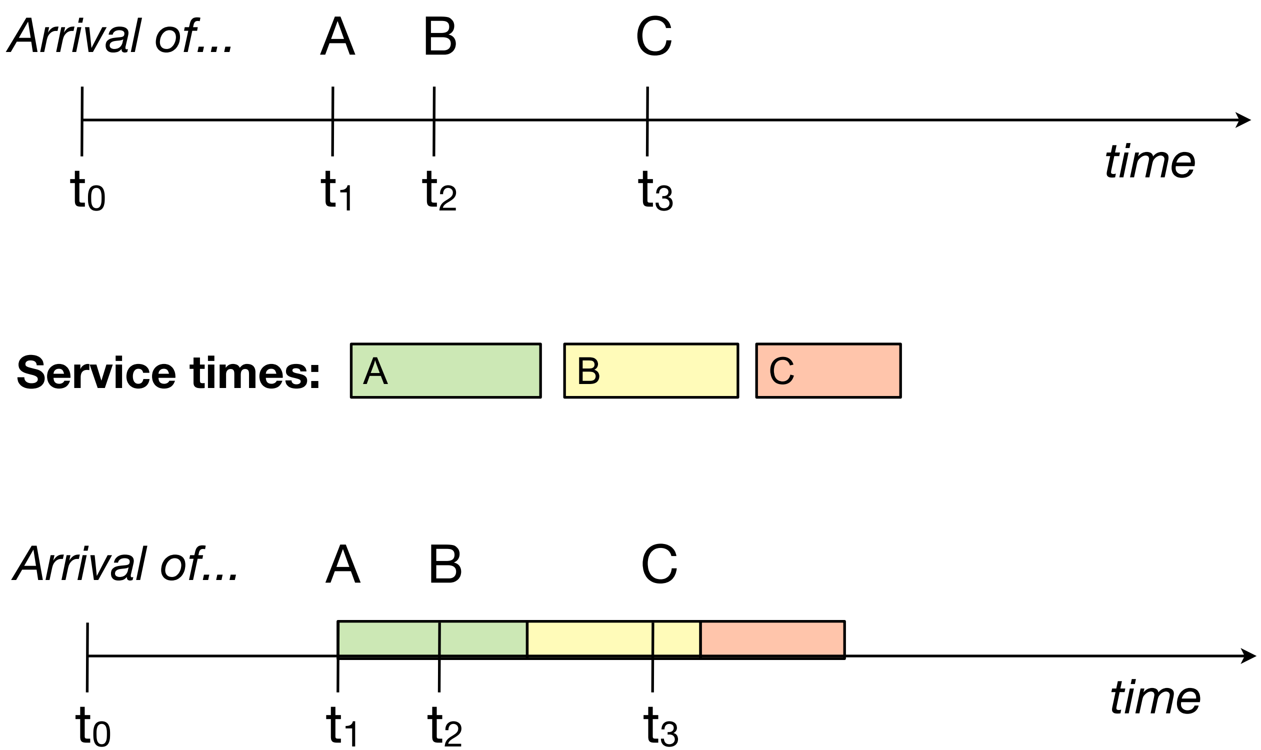
[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Simulating a Queue (2)

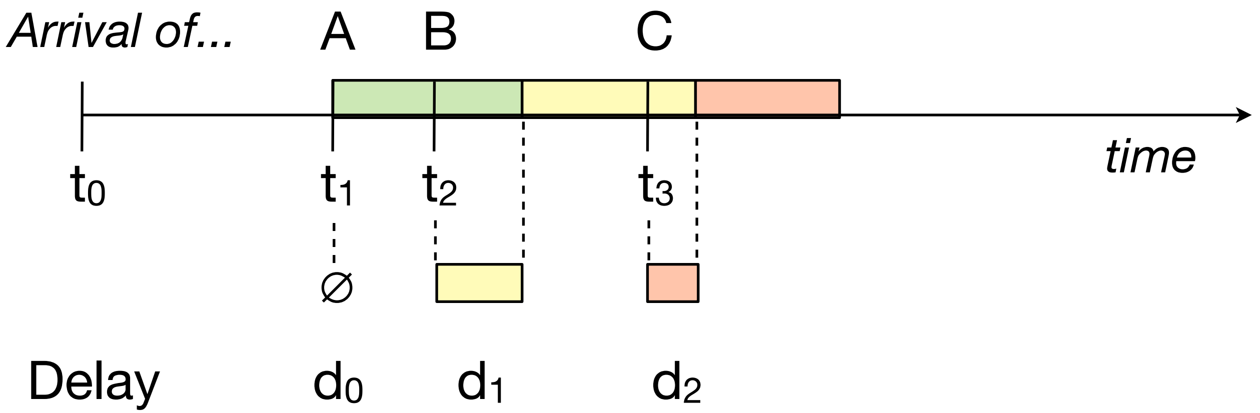


What about service times?

Simulating a Queue (3)



Measuring Waiting Time

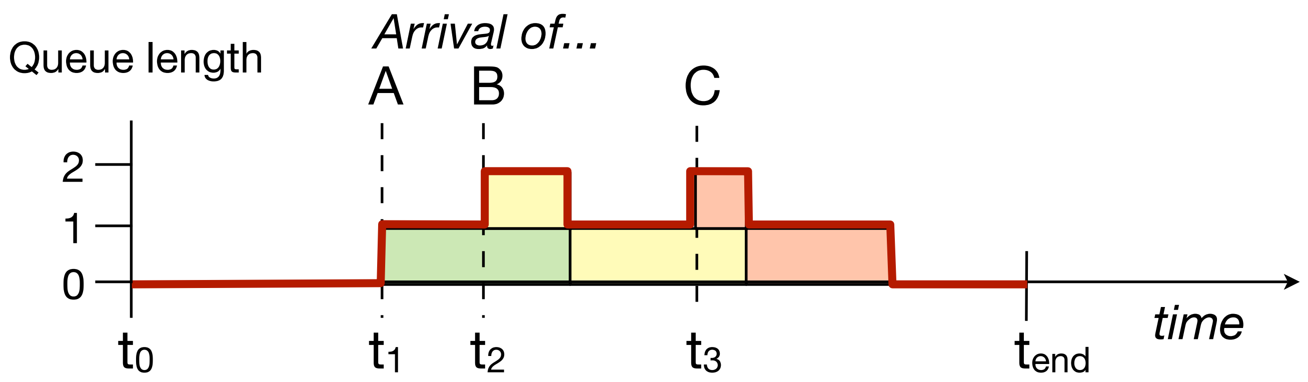


Average waiting time: $1/n \, \Sigma d_i$
(n = number of customers)

Discrete-time metric: Average taken over a discrete set of numbers

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Measuring Queue Length



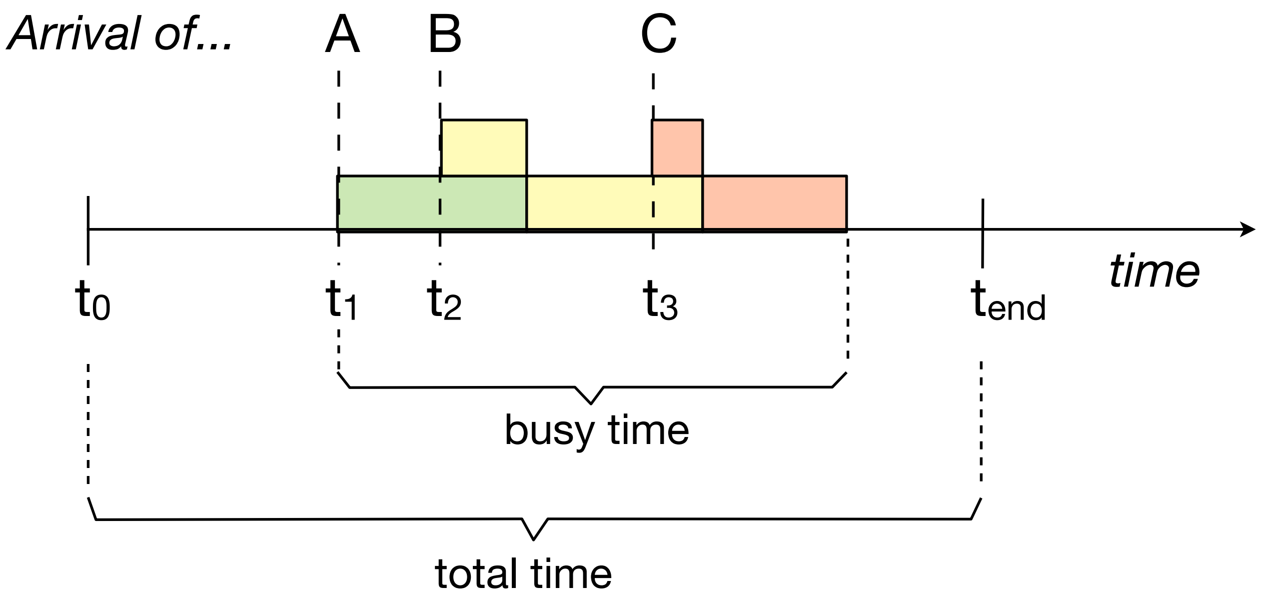
Average queue length:
Area under the red curve / total time

How to compute: Maintain total area in a variable.
Add area since last event when processing a new event.

Continuous-time metric: Average taken continuously over time

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Measuring Server Utilization



Server utilization: Busy time / total time

On the Meaning of Measurements (1)

- **How to interpret these measurements?**
- In the previous example:
  - Per-customer delays are measured in one particular simulation run
  - Different runs, different delays → different results

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

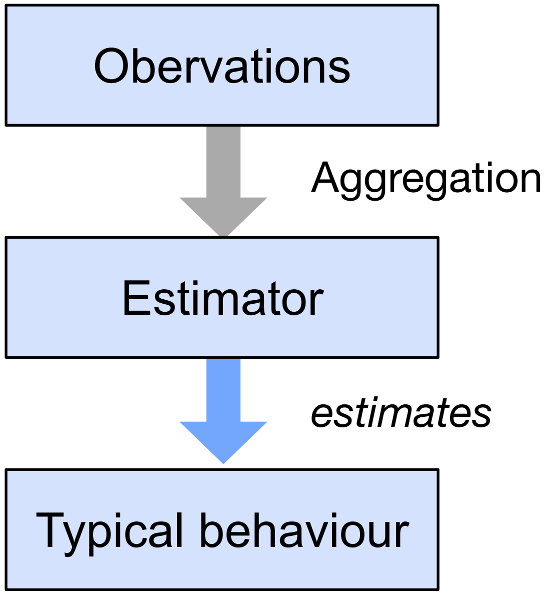
On the Meaning of Measurements (2)

- **Why use aggregated information?**
- Aggregated information (here: average) gives a concise description of system characteristics
- Why not consider distributions instead of measured averages?
- In the previous example:
  - The first customer does *never* have to wait (d_0=0), which is not true for the following ones
  - This real behaviour is not covered by a distribution

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

On the Meaning of Measurements (3)

- Goal: Extract the **“truly typical” behaviour** of the model
- Simulation runs give only an **estimator** for this behaviour



[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Lessons learned

- Simulation means to model a system and run it in an artificial environment
- **Parameters** select system behaviour
- **Metrics** characterize a system
- Discrete event models do not only describe discrete time systems.
- Stochastic models are often used when mathematical models are intractable.

Implementing a Simulation Model

- **Overview:**
  - Next-event time advance algorithm
  - Maintaining the future event set
  - Adding statistics
  - Randomness
  - Object-oriented implementation
  - Race conditions

Next-event time advance algorithm

- **Basic algorithm**

```
initialize;

while (stopping condition is false) {
    get next event from future event set
    advance time to this event
    process event
    generate new event(s) and add them
           to the future event set
}
```

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Simulating the Queue (1)

- **State of the model**:
  - Simulation time
  - Queue for storing the tasks (FIFO)
  - Server state: idle or busy
  - Events: arrival and departure.
    Variables:
    - Time of next arrival
    - Time of next departure
- Arrivals and departures are modeled as a Poisson process.
  Inter-arrival times follow an exponential distribution

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Simulating the Queue (2)

- **Processing events:**
  - on arrival: add task to queue, schedule departure
  - on departure: remove task from queue
- **Initialization:**
  - generate first arrival time
- **Setting the server state**
  - If a task arrives, the server is busy
    (further tasks have to wait)
  - If the queue is empty after processing, the server is idle

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Simulating the Queue (3)

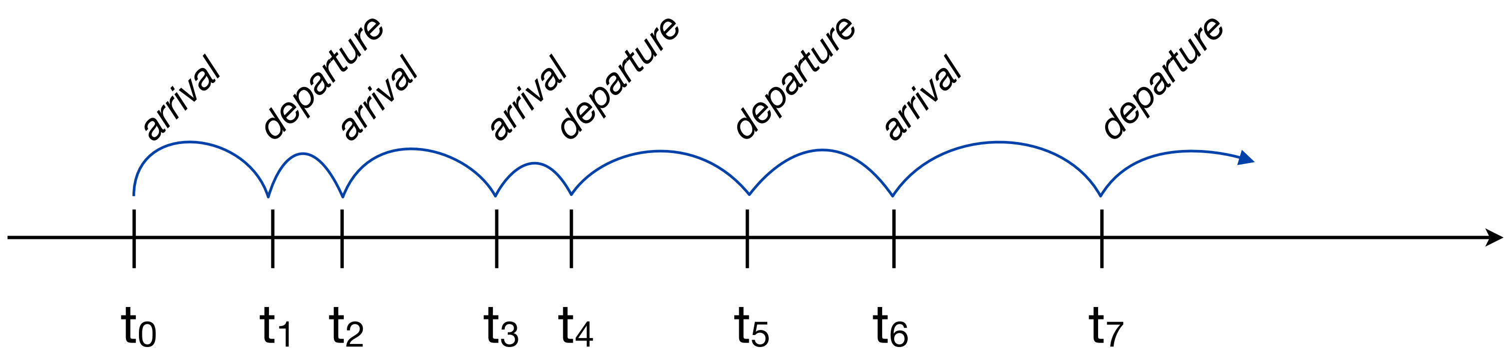
```
server_state = IDLE;
double sim_time = 0.0;
double next_departure = HUGE_VAL; ← set to infinity
double next_arrival = random.poisson(mean_arrival); ← first event

while (sim_time < T_MAX) {
    if (next_arrival < next_departure) {  ← next event: arrival
        sim_time = next_arrival;      ← advance clock
        if (server_state == IDLE) {
            server_state = BUSY;
            next_departure = sim_time + random.exp(mean_processing);
        } else {
            queue.push( new task(sim_time) );
        }
        next_arrival = sim_time + random.exp(mean_arrival);
    } else {     ← next event: departure
        sim_time = next_departure;
        if ( queue.empty() ) {
            server_state = IDLE;
            next_departure = HUGE_VAL;
        } else {
            task t = queue.pop();
            next_departure = sim_time + random.exp(mean_processing);
        }
    }
}
```

cf. [H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Simulating the Queue (4)

- **Next-event time advance:**



[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Simulating the Queue (5)

- **Key techniques used here:**
  - When an arrival event is processed, the next event of the same type is generated (event occurrence is modeled by a Poisson process)
  - Departure events are "*poisoned*" if there is no waiting task (by setting its time to infinity)

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Randomness

- Random numbers are essential to stochastic models
- There are sources of true randomness
  - difficult to produce, difficult to *re*-produce
  - Reproducible simulation results are desired!
- Therefore, **pseudo-random** numbers are used, which can be generated **deterministically**

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Types of RNGs

- Pseudo random number generators (PRNGs) generate sequences of numbers
- PRNGs can only produce a *finite* quantity of different numbers and sequences of *finite* length (until numbers are repeated)

- There are various PRNGs. **Which are good, which are bad?**
  - Good RNGs produce every possible number (or a large fraction) of the value range
  - Good RNGs have a long period
  - Bad RNGs are sensitive to seed selection

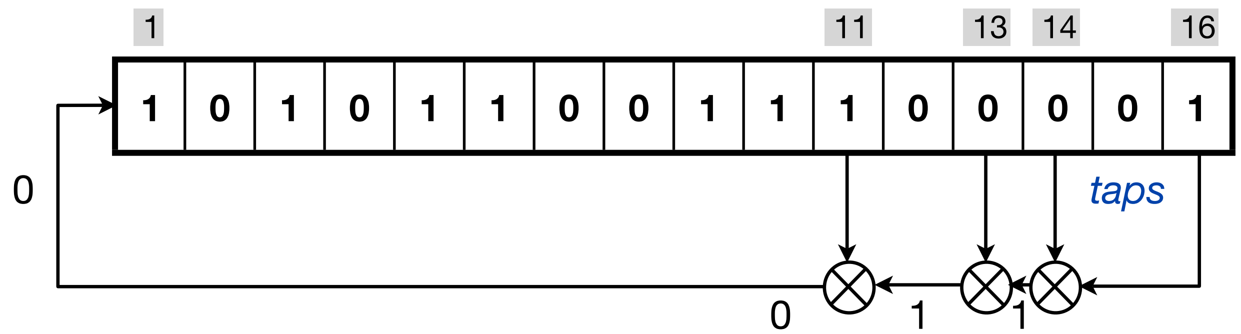
Types of PRNGs: LCG

- **Linear congruential generator (LCG)**:
  - $x_{n+1} = (ax_n + c) \bmod m$
  - Parameters:
    - m: modulus, $m>0$
    - a: multiplier, $0 < a < m$
    - c: increment, $0 \le c < m$
    - x_0: start value or “seed”, $0 \le x_0 < m$
- **Example:** $x_{n+1} = (3x_n + 5) \bmod 8$, $x_0 = 5$
  Sequence: 5,4,1,0,5,4,1,0,5,4,1,0,...

- Easy implementation, but short period and sensitive to parameters
- Used in glibc, ANSI C, MS C++

Types of PRNGs: LFSR

- **Linear feedback shift register (LFSR)**:



feedback polynomial $x^{16}+x^{14}+x^{13}+x^{11}+1$

- Period depends on the feedback function
- There are tables with maximum cycle-length polynomials

Types of PRNGs: LFG

- **Lagged Fibonacci generator**:
  - Fibonacci sequence: $F_n = F_{n-1} + F_{n-2}$, $F_0 = 0$, $F_1 = 1$
  - Generalized form: $F_n = (F_{n-j}\ op\ F_{n-k}) \bmod m$, $0 < j < k$
    op = binary operator
  - Good choices for (j,k) are e.g. (7,10), (5,17),(6,31).
    (see D. Knuth, The Art of Computer Programming, Vol. 2)

- Longer period than LCGs, but sensitive to initialization
- Used in Boost

Types of PRNGs: Mersenne Twister

- **Mersenne Twister** (Matsumoto, Nishimura 1997)
  - Twisted generalized feedback shift register
  - Very long period
  - high order of dimensional equidistribution (low correlation between successive numbers)
  - More complicated to implement
    (look for standard implementation of MT19937)

- Implemented as part of GSL (GNU scientific library)
- **Alternative**: Complementary Multiply-with-carry generator

PRNGs Overview

PRNG	Period
LCG (mod m)	max. m
LFG (mod m, operator = addition)	max. $(2^k-1)*(m/2)$
LFSR (n bits)	max. 2^n-1
Mersenne Twister MT19937 (32 bit)	$2^{19937}-1 = 4.3 * 10^{6001}$
Complementary Multiply-with-carry	10^{410928}

RNGs and Distributions (1)

- PRNGs produce a sequence of (hopefully) uniform distributed numbers.
- Such sequence can be used to obtain other distributions
- Example for generating Poisson-distributed numbers:

```
int rand_poisson(double lambda) {
  double L = e^(-lambda);
  int k = 0;
  double p = 1;
  repeat
      k = k + 1;
      r = uniform random number in [0,1];
      p = p * r;
  until (p <= L)
  return k - 1;
}
```

[Knuth, The Art of Computer Programming, Vol 2., 1969]

RNGs and Distributions (2)

- Generating random numbers an arbitrary distribution with density function *f* and cumulative distribution function *F*
  - **Inverse transform sampling**

    1. Generate a uniformly distributed random number u
    2. Compute x such that $F(x) = u$
    3. return x

  - **Rejection sampling**

    Choose a function *g* such that $f(x) \leq c \cdot g(x)$ for all $x \in \mathbf{R}$ and a constant c

    1. Generate a uniformly distributed random number u
    2. Sample x randomly from $c \cdot g(x)$
    3. If $u < f(x) / c \cdot g(x)$ return x; otherwise goto step 1

RNGs and Distributions (2)

- Generating random numbers for arbitrary distributions
  - Inverse transform sampling

    1. Generate a uniform

  - Rejection sampling

Seed selection

- PRNGs produce sequences deterministically:
  Same seed - same PRN sequence
- Use seeds as simulation parameters
- Don’t use 0 (e.g. LCG with c=0 gets stuck)
- Avoid even numbers (e.g. LCG with c=0 and even m produces fewer different numbers)

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Multiple RNGs (1)

- In the example, the same RNG is used for inter-arrival and processing times

```
[...]
while (sim_time < T_MAX) {
    if (next_arrival < next_departure) {
        sim_time = next_arrival;
        if (server_state == IDLE) {
            server_state = BUSY;
            next_departure = sim_time + random.exp(mean_processing);
        } else {
            queue.push( new task(sim_time) );
        }
        next_arrival = sim_time + random.exp(mean_arrival);
    } else {
        sim_time = next_departure;
        if ( queue.empty() ) {
            server_state = IDLE;
            next_departure = HUGE_VAL;
        } else {
            task t = queue.pop();
            next_departure = sim_time + random.exp(mean_processing);
        }
    }
}
```

Multiple RNGs (2)

- Using the same PRNG can lead to correlations between inter-arrival and processing times.

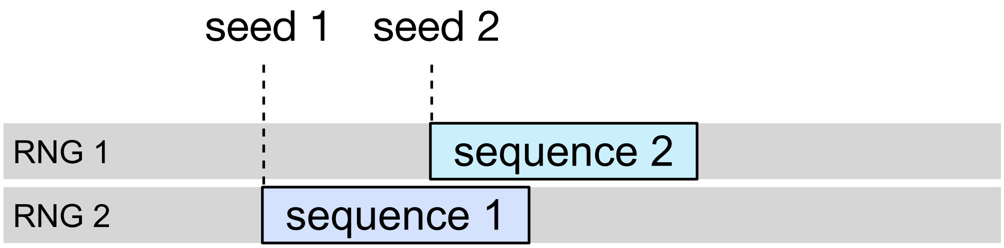
- Here, arrival and processing times are independent random variables.
- Initialization with different parameters should be possible

- **General recommendations:**
  - Use different PRNGs for different random variables
  - Use different seeds!

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Multiple RNGs (3)

- Overlapping sequences should be avoided:



- **More recommendations:**
  - Ensure that seeds separate the sequences
  - Limit simulation length

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Adding Statistics (1)

```
[...]
double last_event_time = 0.0;
waiting_time_total = 0.0;

while (sim_time < T_MAX) {
    if (next_arrival < next_departure) sim_time = next_arrival;
    else sim_time = next_departure;
    update_statistics();
    if (next_arrival < next_departure) {
        if (server_state == IDLE) {
            server_state = BUSY;
            next_departure = sim_time + rng2.exp(mean_processing);
        } else {
            queue.push( new task(sim_time) );
        }
        next_arrival = sim_time + rng1.exp(mean_arrival);
    } else {
        if ( queue.empty() ) {
            server_state = IDLE;
            next_departure = HUGE_VAL;
        } else {
            task t = queue.pop();
            next_departure = sim_time + rng2.exp(mean_processing);
            waiting_time_total = sim_time - t.arrival_time;
        }
    }
    last_event_time = sim_time;
}
```

Adding Statistics (2)

- Order of execution: Advance clock, record statistics, process events (state changes)

```
void update_statistics() {
  double time_since_last_event = sim_time - last_event_time;

  cumulated_queue_length += queue.length() * time_since_last_event;

  if (server_state == BUSY)
    busy_time_total += time_since_last event;
}

[...]

// at the end of the simulation:
avg_queue_length = cumulated_queue_length / sim_time;
avg_utilization = busy_time_total / sim_time;
```

Recording statistics

- Simulation parameters for the queueing example:
  - RNG parameters: seeds and mean values (mean inter-arrival time A and mean service/processing time S)
  - Simulation duration (time limit or max number of tasks)

Results for 1000 tasks, A = 1, S=0.5

Seed A	**Seed S**	**Avg. Utilization**	**Avg. Queue Length**	**Avg. Waiting Time**
23	17	0,534	0,572	0,563
25	89	0,496	0,478	0,479
167	11	0,505	0,458	0,453
235	21	0,506	0,451	0,435

Simulations performed with example program in:
[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Processing statistics

- Average over different simulation runs:

Seed A	Seed S	Avg. Utilization	Avg. Queue Length	Avg. Waiting Time
23	17	0,534	0,572	0,563
25	89	0,496	0,478	0,479
167	11	0,505	0,458	0,453
235	21	0,506	0,451	0,435
Average:		0,510	0,490	0,483
Standard error:		0,008	0,028	0,028

Simulations performed with example program in:
[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

The Impact of Simulation Time (1)

- Example: Simulating a busy server
  Mean inter-arrival time A = 1, mean service time S = 0.9
  Seeds A,S = 23,17
- Analytical results available (M/M/1 queue)

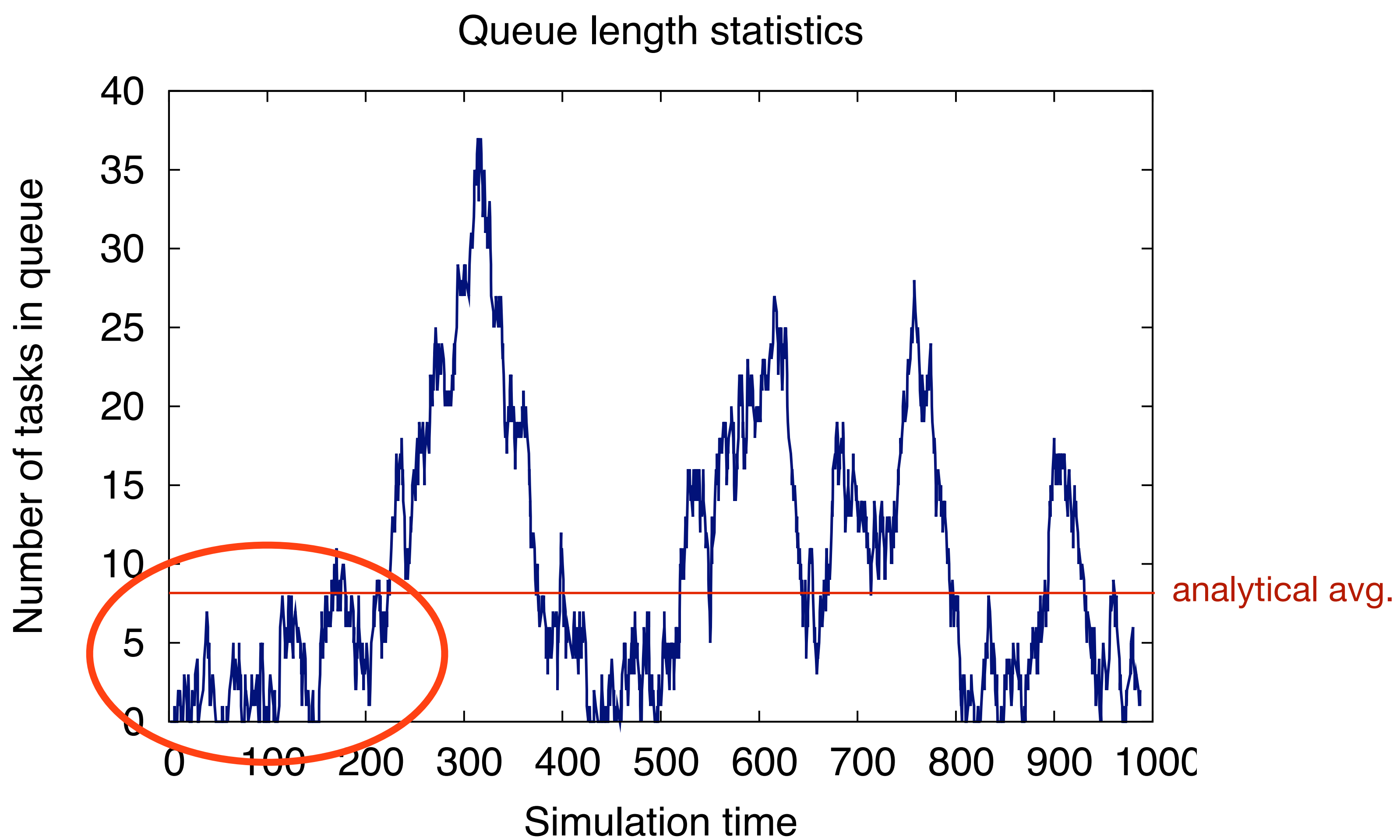
Number of tasks	**Avg. Utilization**	**Avg. Queue Length**	**Avg. Waiting Time**
10	0,563	0,490	0,585
100	0,849	1,885	2,093
1000	0,957	9,965	9,839
10000	0,913	8,459	8,496
Analytical:	0,9	8,1	8,1

Simulations performed with example program in:
[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

The Impact of Simulation Time (2)

- For short simulations (small number of tasks) the results highly deviate from analytical values.
- Long simulation runs seem to produce better results.
- What is the reason for this behaviour?
- We observe the queue length over time...

The Impact of Simulation Time (3)



[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

The Impact of Simulation Time (4)

- At the beginning the queue is empty
- It needs some time to fill the queue, a high number of waiting tasks is unlikely
- This initial phase (*initial transient*) should be removed, if the typical steady-state behaviour is to be observed

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Object-oriented design (1)

- Separate *Modules*
  - Objects which are simulated
  - Event dispatcher to handle events
  - Load generator
- Modules are extended (communicating) FSMs.
  They communicate *only* by message exchange.
- Strict separation of simulation engine from problem-specific modules and events

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Object-oriented design (2)

- Data structure for the future event set
  - Priority queue
  - Events are sorted according to their activation time, next event = minimum key element
  - Which data structures are efficient?
    Heap structures, e.g. Fibonacci heaps, provide quick access to the minimum element and efficient insertion
- Implementation: exercise!

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Avoiding Race Conditions

- Handling simultaneous event arrival:
  - Priorities for different event classes
  - Maintaining the order (according to creation)
  - Testing conditions and subsequent state changes as atomic sequence

- Determinism should always be ensured!

Network Simulation

- A typical simulation setup:
- Protocols are Communicating FSMs
- They are fed by a load generator
- The simulation engine passes messages between the modules and triggers timer events
  - Core modules: Event dispatcher and event queue (future event set)

Simulation Model (1)

- **System Model:**
  - describes the composition of the whole system into modules (and submodules)
    - Module parameters: queue size, processing delay
  - Modules communicate by message exchange over links
    - Link parameters: delay, bandwidth limitation

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

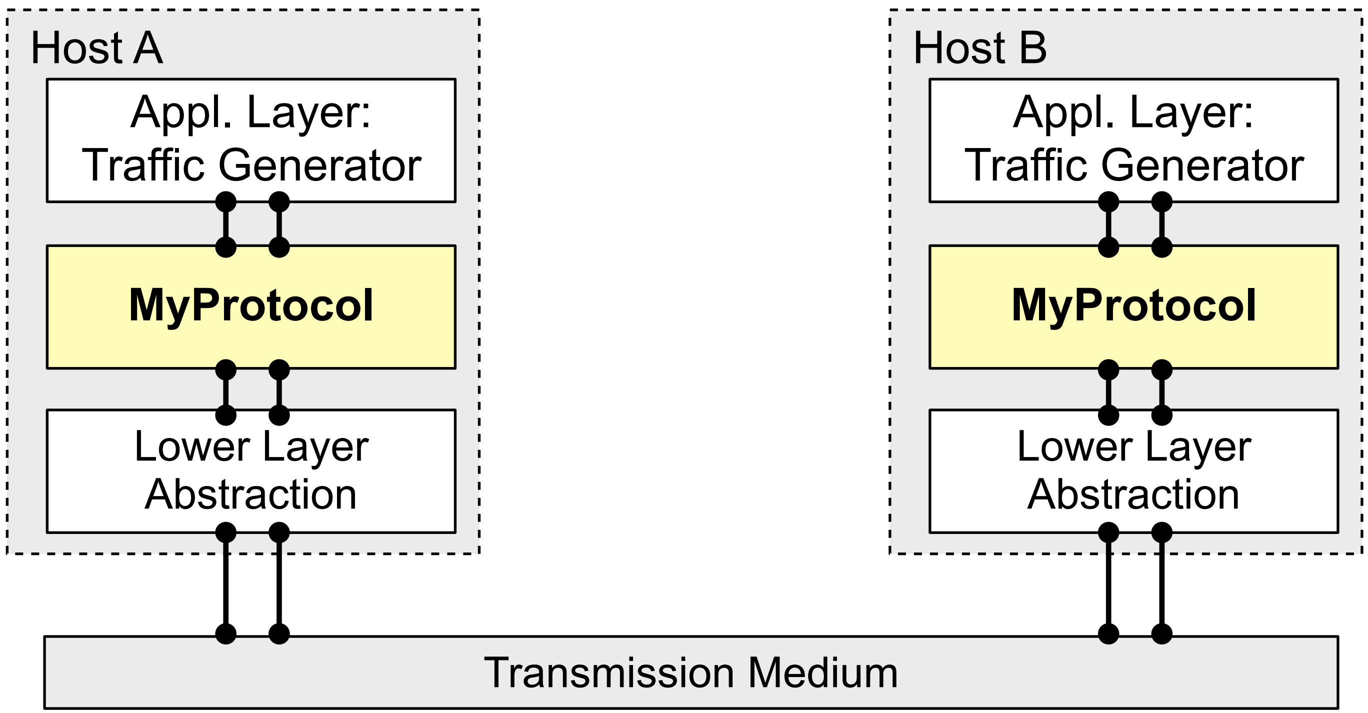
Simulation Model (2)

- **Load Model:**
  - describes the pattern of requests made to the system, e.g. how often are messages injected by an application?

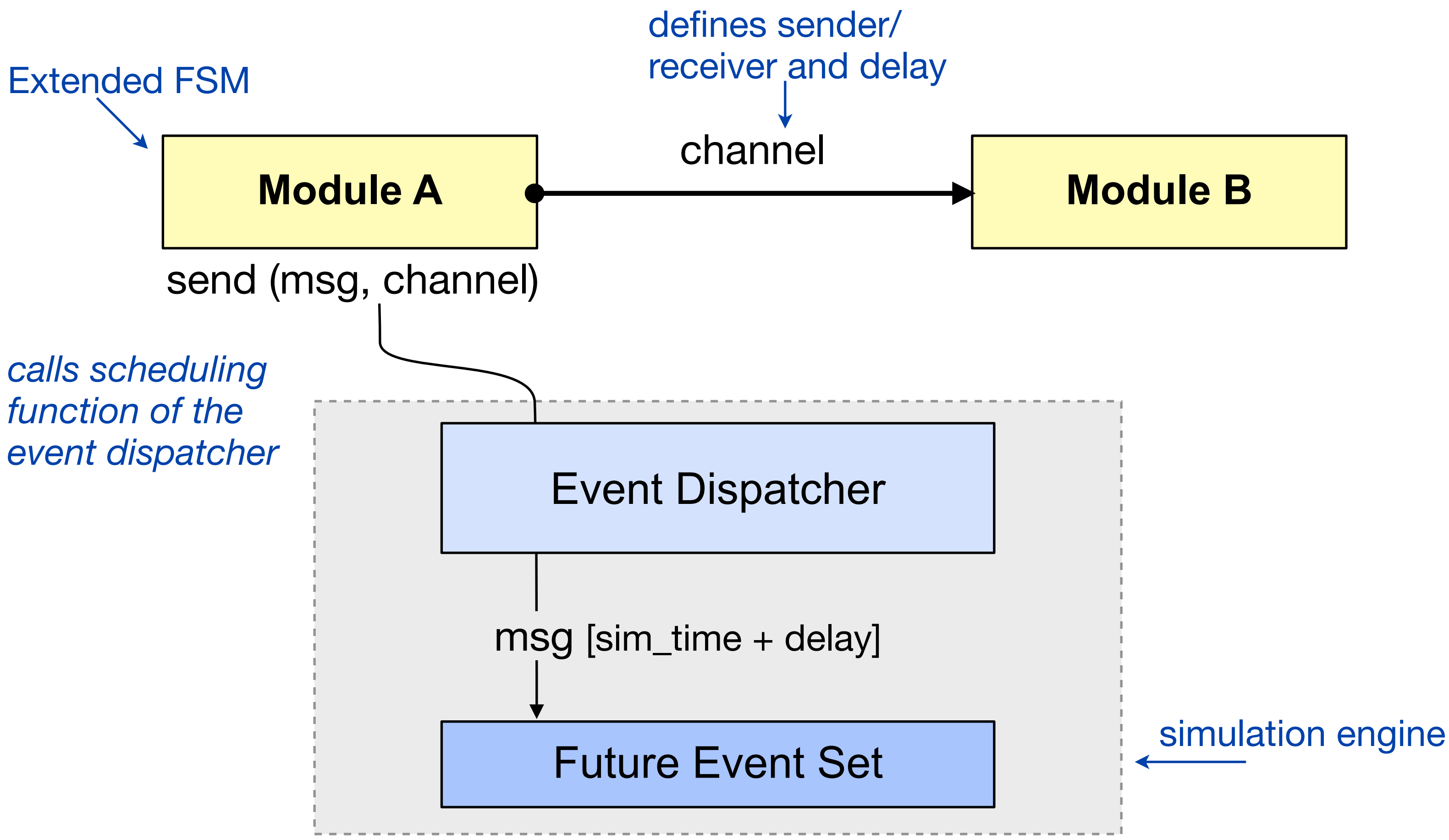
- **Fault Model:**
  - describes the deviation from normal behaviour, e.g. module failures, lossy links

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

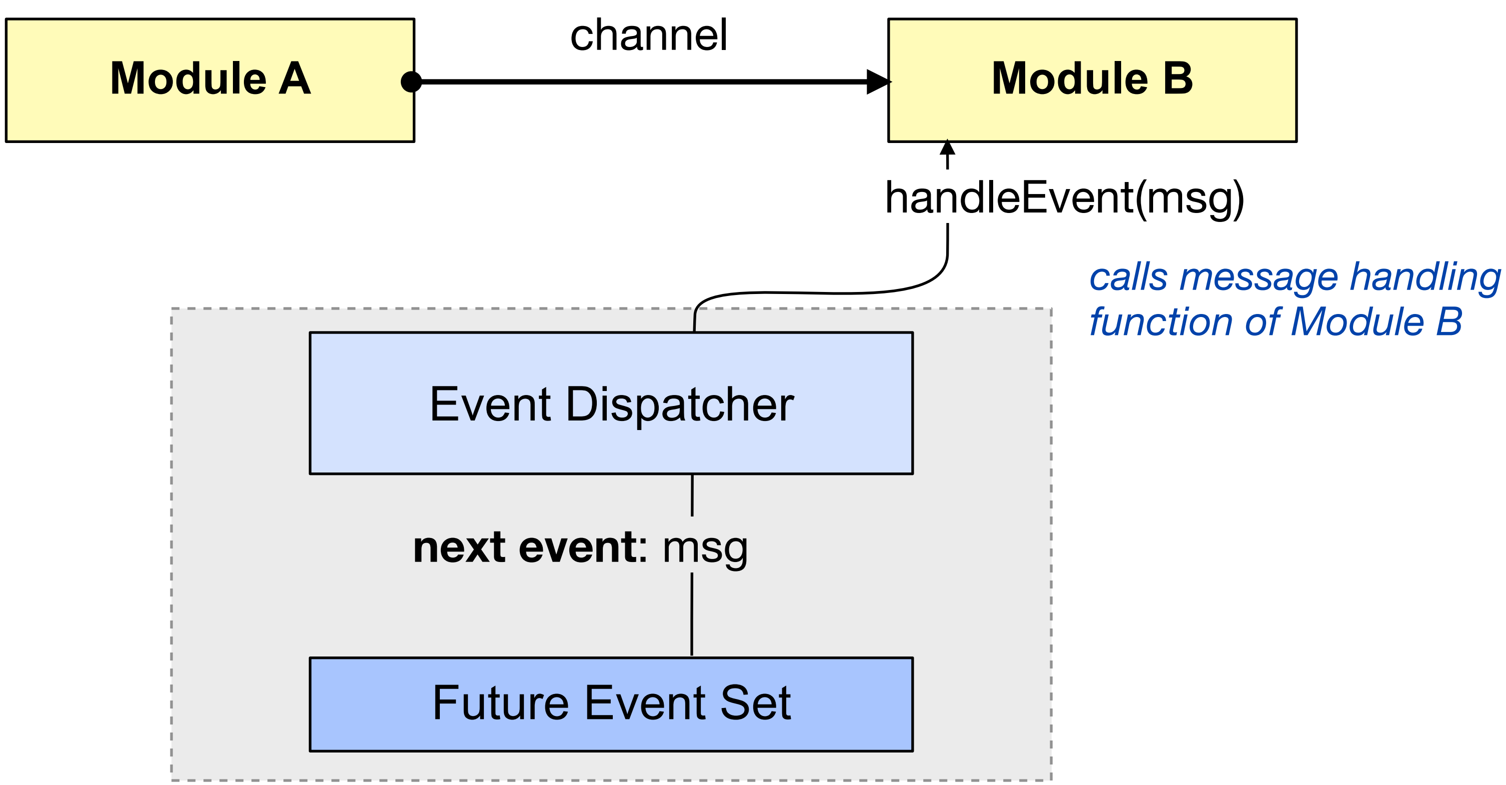
A Typical System Model



Message Passing in Simulation



Message Passing in Simulation



Generating Load

- **Sources of load patterns**
  - Traces (recorded data)
  - Empirical distributions
  - Models for arrival processes

Traces

- Using recorded data of real systems in simulations
  - recorded arrival and service times of a queueing system, recorded packet loss, mobilitiy traces, etc.

- Can be used to extract “typical” behaviour

- *Advantage*: true behaviour, high level of detail, can be used to validate simulation model

- *Disadvantage*: small amount of data, generalization difficult

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

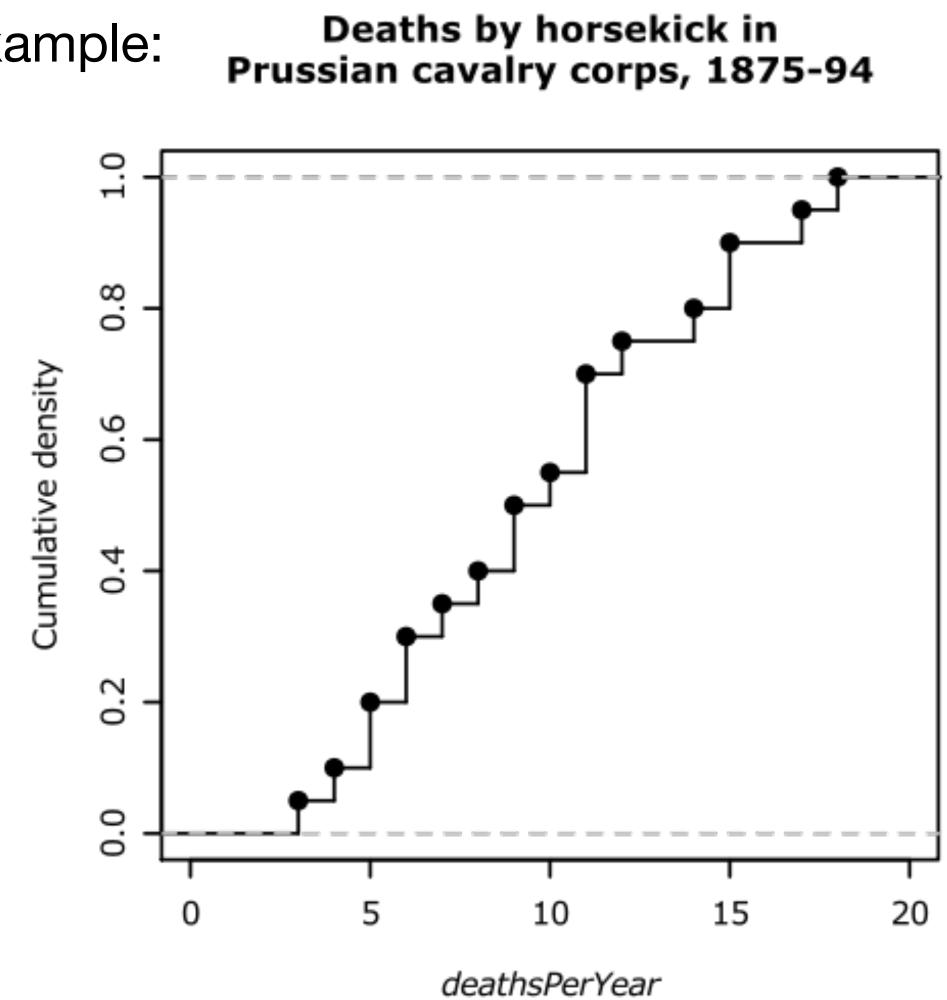
Empirical Distributions

- Generalization of recorded data
  - Assumption: samples follow the same distribution function
  - Let $x_1 \ldots x_n$ be a set of characteristic values with their frequencies h_i taken from a sample
  - Empirical distribution function (ECDF) for the sample:

$$F(t) := \begin{cases} 0, & \text{if } t < x_1 \\ \sum_{j=1}^{i} h_j, & \text{if } x_i \leq t < x_{i+1},\ i \in \{1, \ldots, k-1\} \\ 1, & \text{if } x_k \leq t \end{cases}$$

Empirical Distributions

- Example:



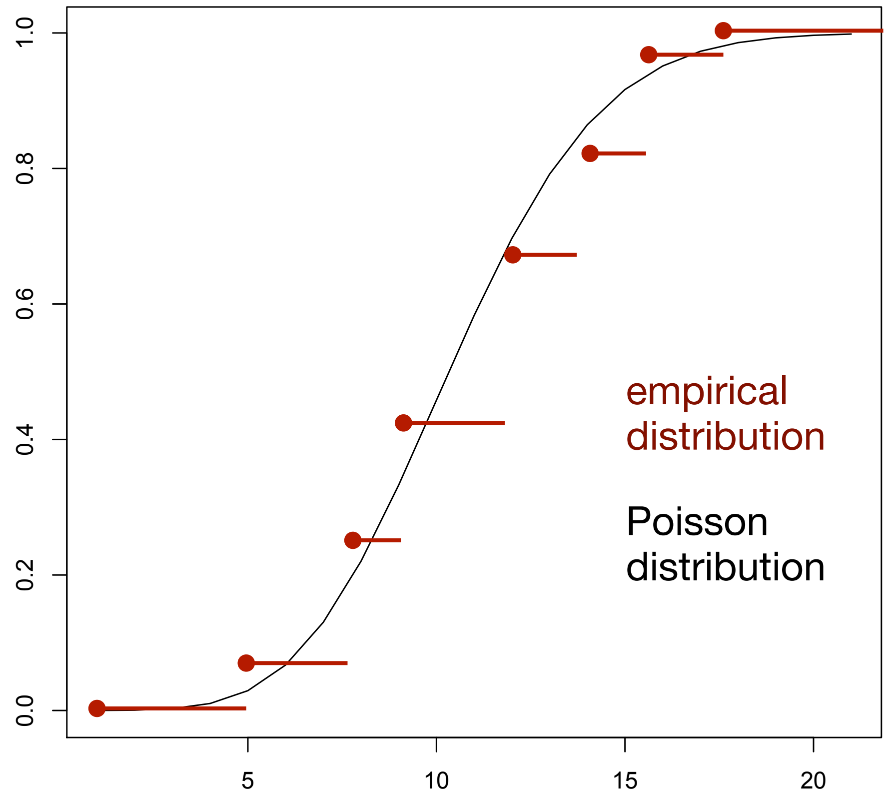
Empirical Distributions

- Sometimes not all characteristic values can be observed
  ... but it is known how many observations fall into a certain interval
- Data can be grouped into intervals
- Distribution function can be defined as follows:
  - interval borders define points of the function
  - interpolate between the points

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Analytical distributions (1)

- Empirical distributions can be approximated by analytical distributions



Analytical distributions (2)

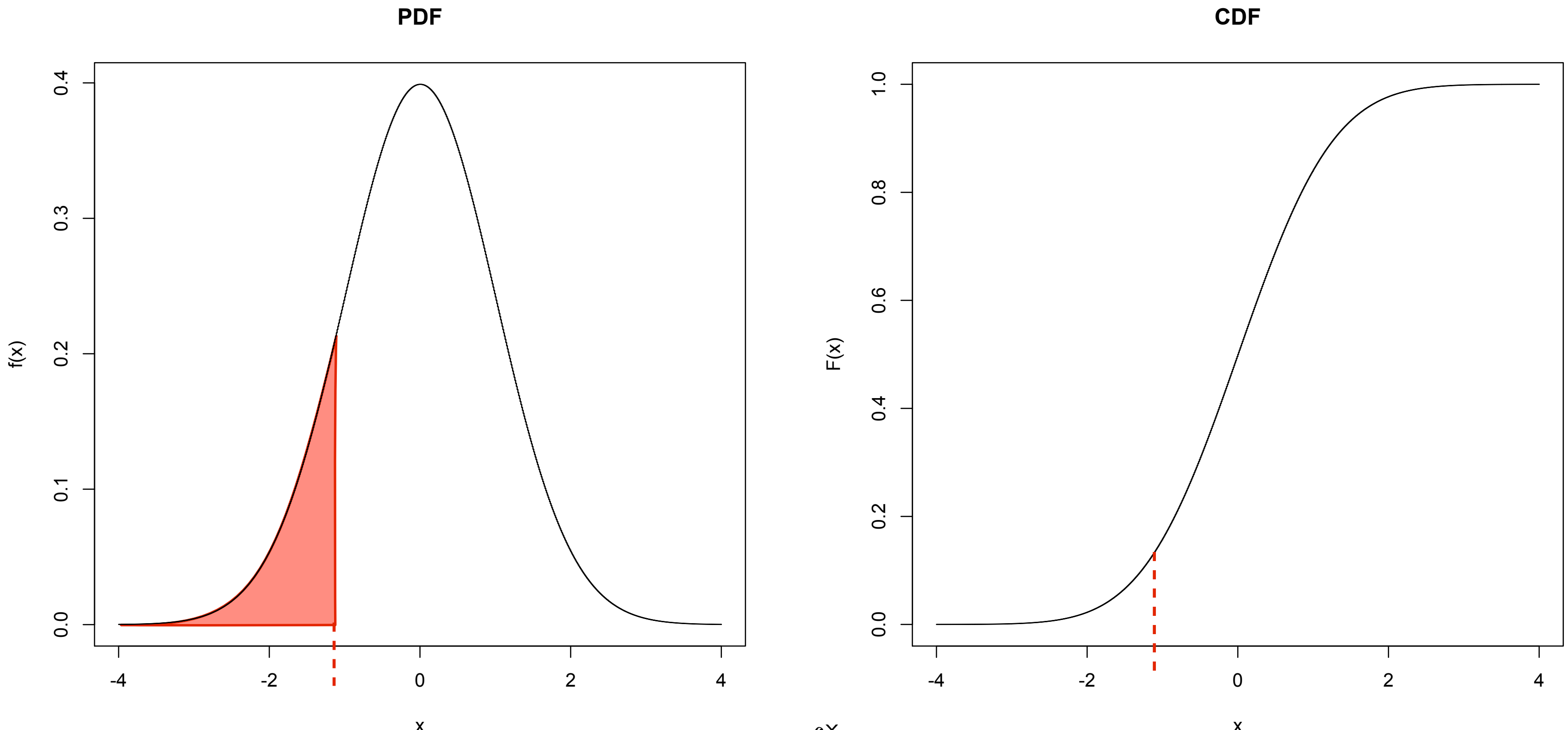
- How to choose a distribution function that matches the empirical distribution?
  - Similarity of the diagrams (ECDF plot and analytical distribution function)
  - Knowledge about the process or the type of events (e.g. independent arrivals are modeled best by a Poisson process)
  - Quality criterion: Goodness of fit test

Probability Distributions Revisited (1)

- **Let X be a random variable**
- **Cumulative distribution function (CDF):**
  - $F(x) = \text{Prob}[X \leq x]$
  - Range of values: [0,1]
  - Monotonically increasing
- **Probability density function (PDF):**
  - $f(x) = \text{Prob}[X = x]$

Probability Distributions Revisited (2)

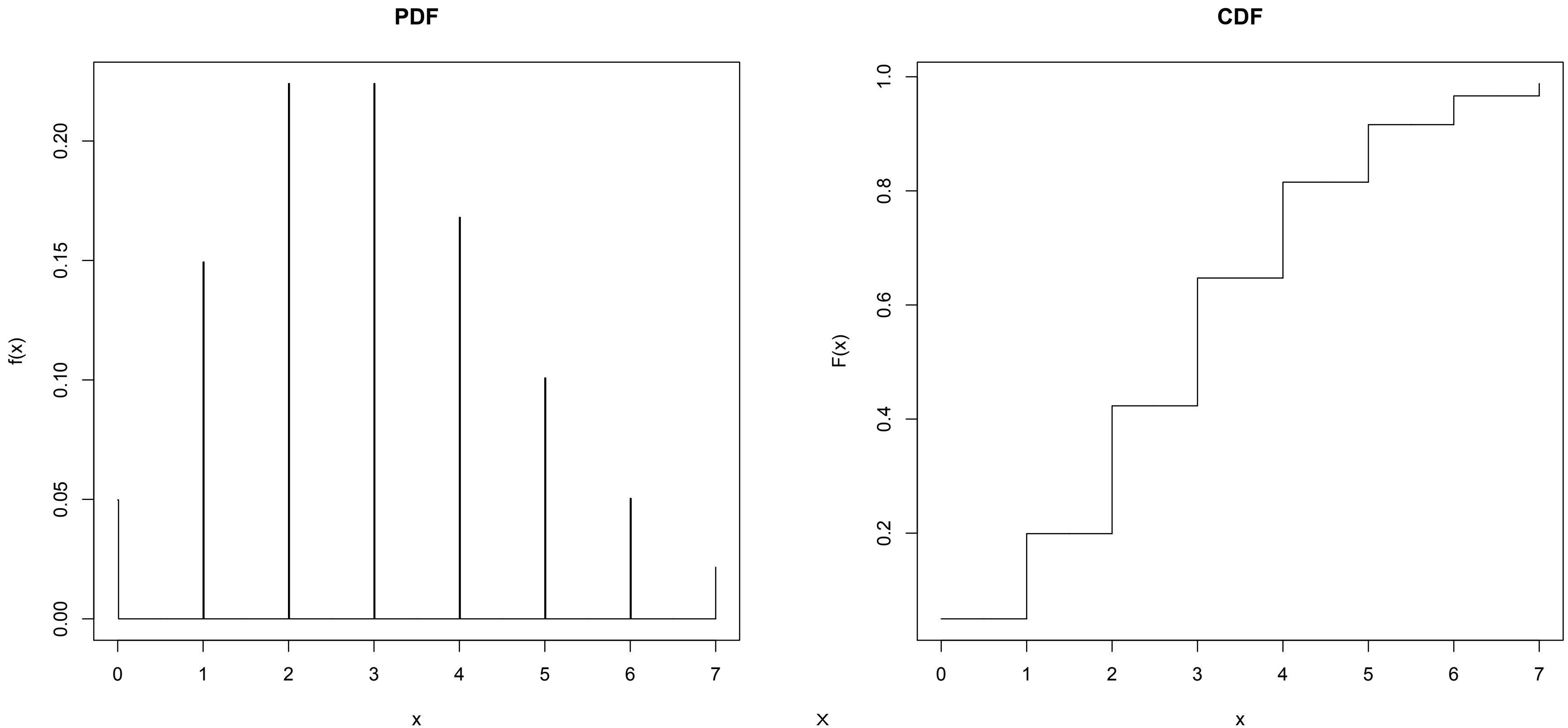
- Example: Normal distribution



$$F(x) = \int_{t=0}^{x} f(t)$$

Probability Distributions Revisited (3)

- Example: Poisson distribution with λ=3



$$F(x) = \sum_{t=0}^{x} f(t)$$

A Few Discrete Distributions

Name	Parameters	Interpretation
Uniform	[a,b]	All outcomes in the interval [a,b] have the same probability
Bernoulli	p	“coin flip” with success probability p
Binomial	n, p	Number of successes of *n* independent Bernoulli trials with success probability p
Geometric	p	Number of successive failures of independent Bernoulli trials until the first success
Poisson	λ	Number of events within a unit time interval, if the time between event arrivals is exponentially distributed with rate λ.

A Few Continuous Distributions

Name	Parameters	Interpretation
Uniform	[a,b]	All outcomes in the interval [a,b] have the same probability (usually the interval is [0,1])
Exponential	λ	Time until the next arrival of an event, if events arrive independently at a rate of λ events per unit time (cf. Poisson)
Normal	μ, σ	The mean of independent repetitions of *any* random experiment converges towards the normal distribution (mean μ, standard deviation σ).
Pareto	α	Can be used to describe the time between event occurrences (cf. Exponential)

Goodness of Fit

- How well do two distributions match?
  - Compare distributions graphically
  - Goodness-of-fit tests
    - χ^2 test (chi square test)
    - Kolmogorov-Smirnov test
    - ...
  - cf. Exercise 8

Chi Square Test (1)

- Comparison of the empirical histogram with a conjectured distribution
- Null Hypothesis: Sample data follows a certain distribution
- Method:
  - Divide the observations into k cells
  - Calculate the frequency f_i of observations in each cell
  - Calculate the expected frequency $n \cdot p_i$ according to the conjectured distribution
  - Calculate chi square: $\chi^2 = \sum_{i=1}^{k} \frac{(f_i - np_i)^2}{np_i}$
  - Reject the hypothesis, if χ^2 is too large

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Chi Square Test (2)

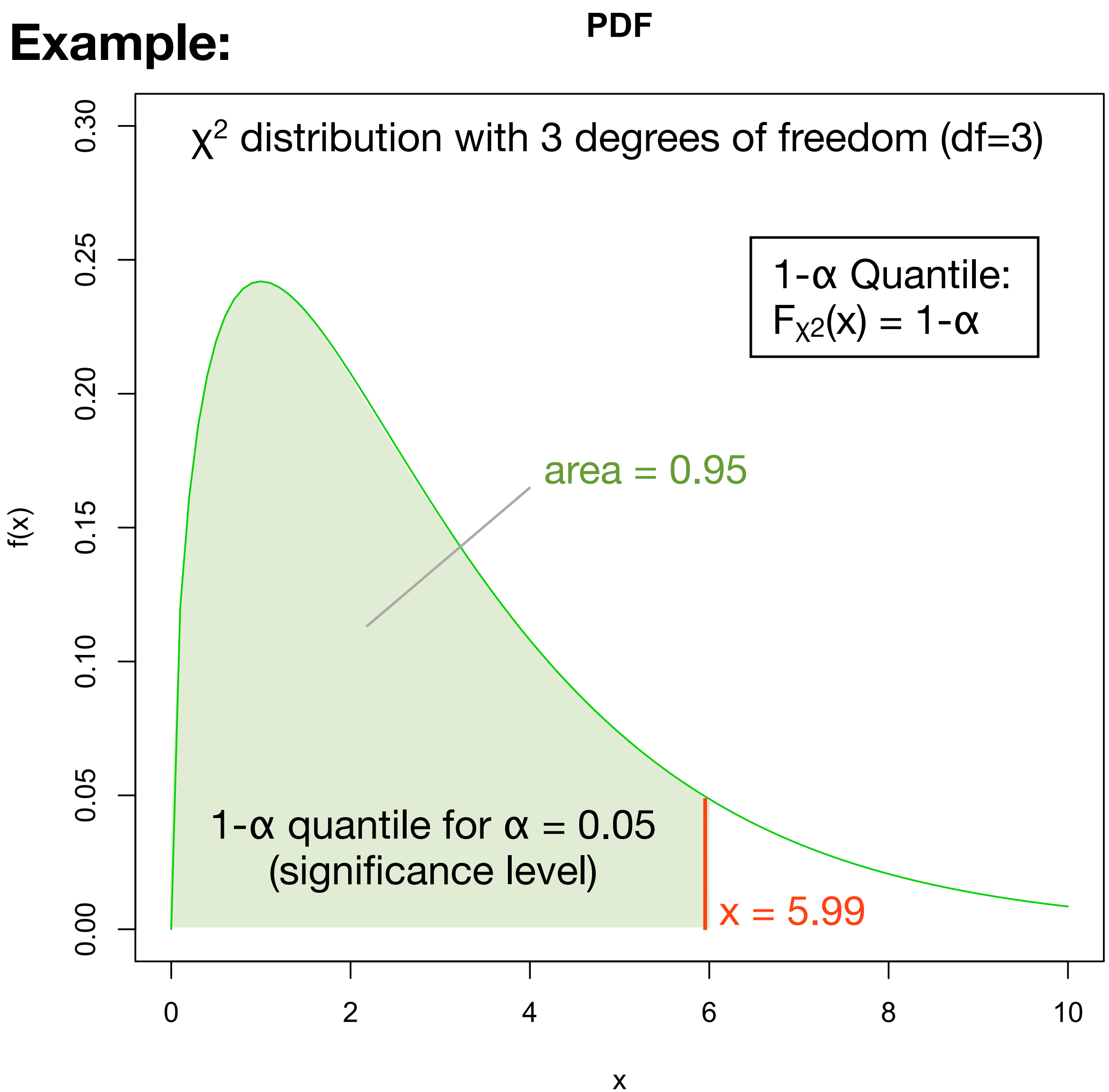
- The test statistics is assumed to be chi-square distributed with k-1 degrees of freedom.
- For a significance level α, the hypothesis should be rejected if $\chi^2 >$ $(1-\alpha)$-quantile of the χ^2 distribution with k-1 degrees of freedom.
- $\chi^2_{k-1,1-\alpha}$ can be looked up from a table
- Statistics programs calculate the significance (p-values)

- Rule of thumb for choosing intervals:
  at least 3 intervals; $n \cdot p_i \geq 5$ for most of the intervals

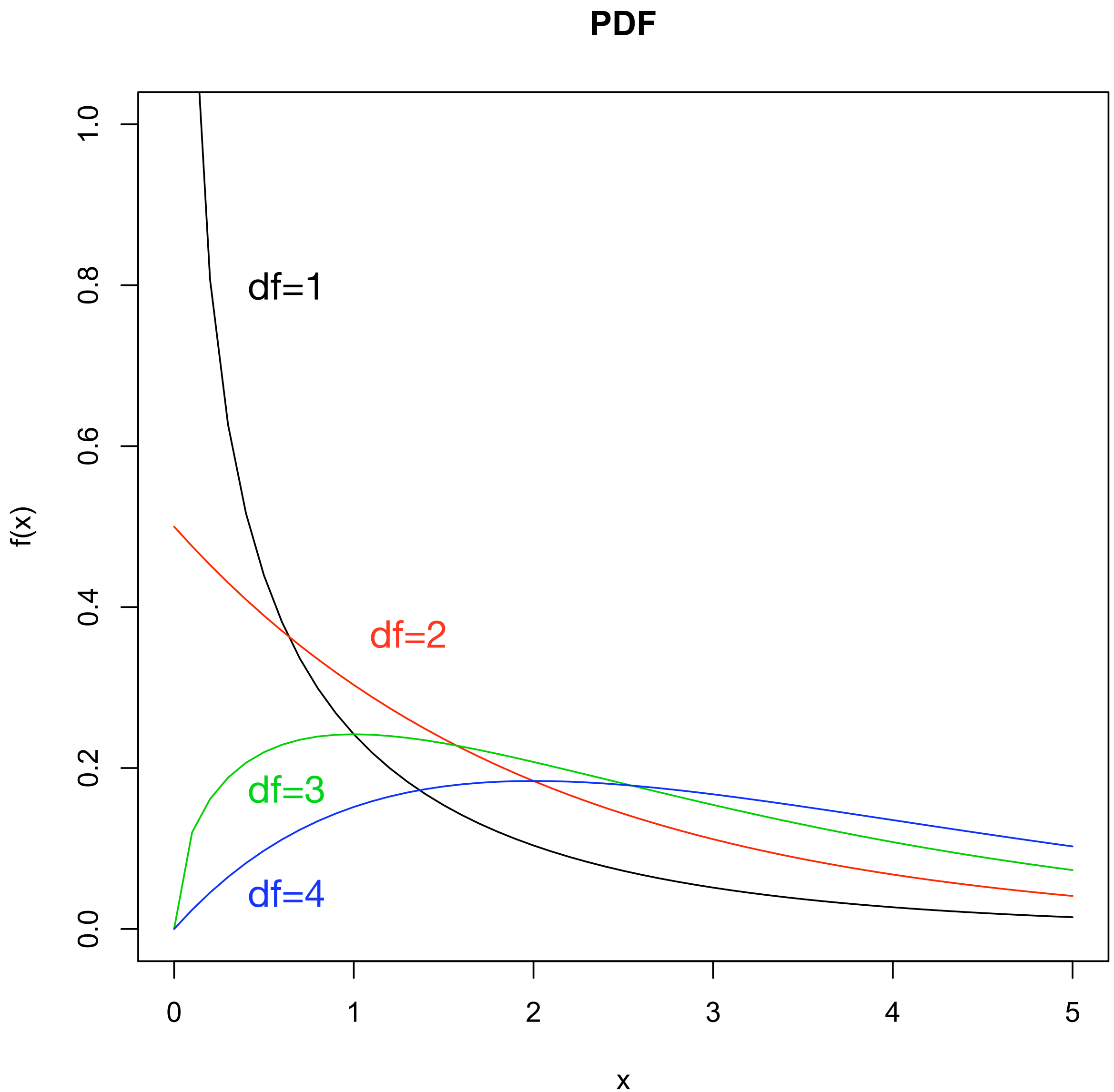
[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Quantile of the χ^2 Distribution

Example:



The χ^2 Distribution



P-values

- P-value describes the significance of a test result
  - Probability of getting a result that is at least that extreme (all under the assumption of a null hypothesis)
  - The lower the P-value the less likely the result

- **Example**: Chi square test for a coin flip.
  - Null hypothesis: the coin is fair
  - 20 heads, 20 tails: $\chi^2 = 0$, p-value = 1
  - 25 heads, 15 tails: $\chi^2 = 2.5$, p-value = 0.1138
  - 30 heads, 10 tails: $\chi^2 = 10$, p-value = 0.001565

Kolmogorov-Smirnov Test (1)

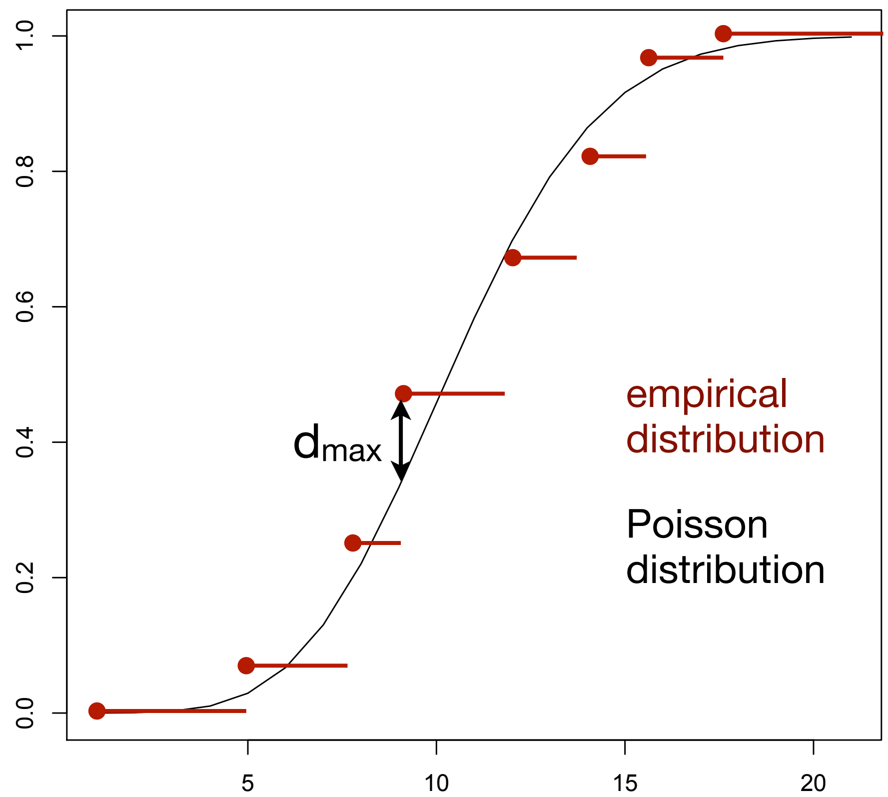
- Comparison of two distributions
  - sample distribution and analytical distribution (one-sample K-S test)
  - two sample distributions (two-sample K-S test)

- Calculates the maximum distance between empirical and conjectured distribution
- Reject hypothesis, if $d_{max} > d_{\alpha}$
  (d_{α} can be looked up from a table)

- Mainly used for continuous distributions

Kolmogorov-Smirnov Test (2)

- Result of the K-S test: Maximum difference in value



Models for Arrival Processes

- At what time does a request arrive in the system?
  - customers, packets, ...

- **Arrival Process = stochastic description of arrivals**

- Some types of arrival processes
  - Constant bit rate (CBR)
  - Renewal process
  - Markov process

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Constant Bit Rate

- **CBR:** Simple model for regular events
- **Characteristics:**
  - Generate load at fixed time intervals
  - Workload per task can be varied

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Renewal Processes

- When interarrival times are independent and identically distributed random variables

- **Special cases:**
  - Poisson process: inter-arrival times are exponentially distributed (continuous time)
  - Bernouilli process: inter-arrival times are geometrically distributed (discrete time)

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Poisson Process (1)

- Discrete number of events, continuous time
- Event occurrences are independent
- Number of events in a time interval of length t follows a Poisson distribution:

$$\mathsf{Pr}_{\lambda,\mathsf{t}}[\mathsf{X} = \mathsf{k}] = \frac{\lambda^{\mathsf{k}}}{\mathsf{k}!}\, \mathrm{e}^{-\lambda \mathsf{t}}$$

- λt = expected number of occurences in time interval [0,t]

Poisson Process (2)

- Time between event occurences (inter-arrival time) is **exponentially distributed:**
- Let X be a Poisson random variable with parameters λ and t, and A a random variable that describes the time until the next arrival. An inter-arrival time $A > t$ is equivalent to $X = 0$ for the interval $[0,t]$.
  - $Pr[A>t] = Pr[X=0] = e^{-\lambda t}$
  - $Pr[A \leq t] = 1 - e^{-\lambda t} =: F(t)$ (cumulative dist. function of A)
  - $f(t) = F'(t) = \lambda e^{-\lambda t}$ (probability density function)
- A follows an exponential distribution

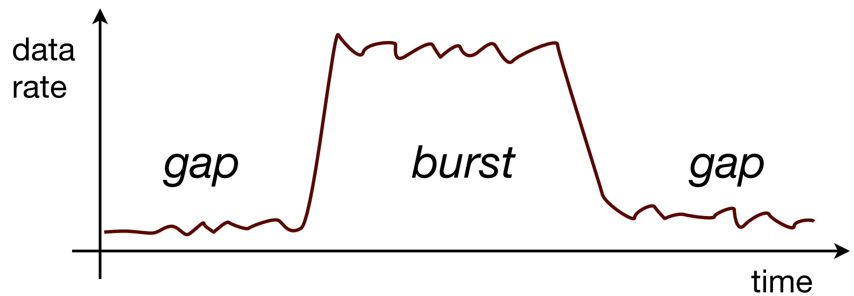
Markov Processes (1)

- Similar to renewal processes, but with history, i.e. there are interdependencies between inter-arrival times
  - State-based
  - Probability of state transitions (defined in a prob. matrix)
  - Transitions depend *only* on the current state.
  - Continuous time: A Markov process remains in a state i and holds this state for a random exponentially distributed time. Then it switches to state j with probability p_{ij}.
  - Discrete time: State transition after each time step (*Markov chain*)

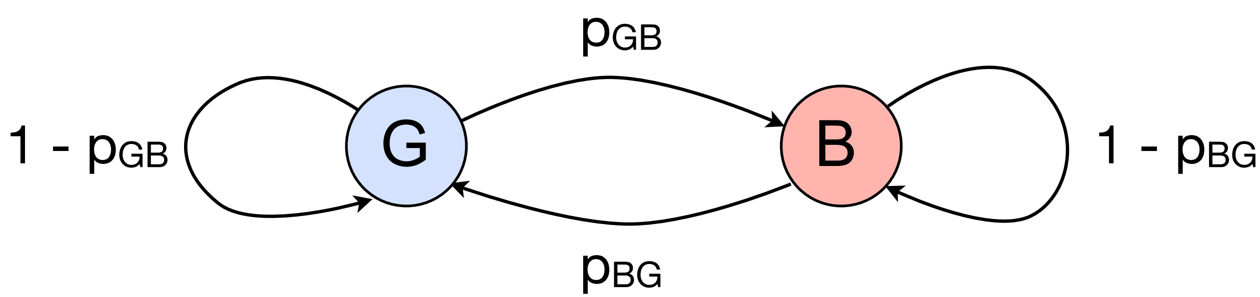
[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Markov Processes, Example (1)

- **Example**: Modeling gaps and bursts of a video stream (load model)



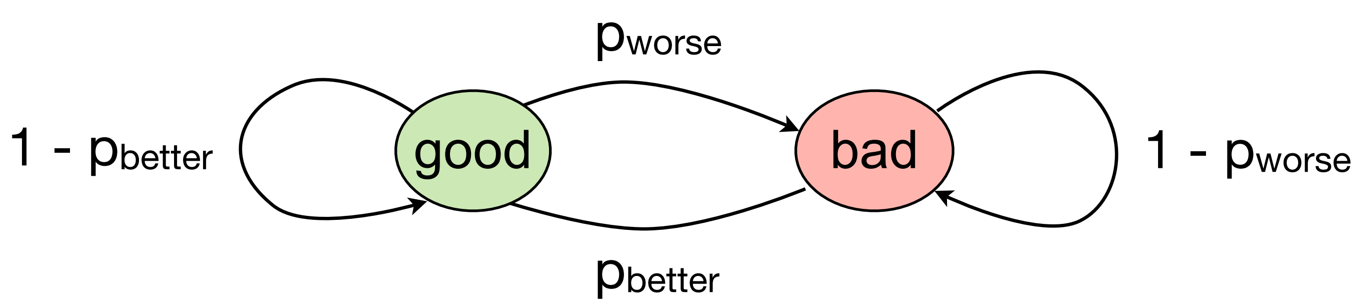
Gap: no packet injection or low arrival rate process
Burst: CBR or high arrival rate process



p_{GB} = probability to switch to the burst state
p_{BG} = probability to switch to the gap state

Markov Processes, Example (2)

- **Gilbert-Elliott Model for bit errors** (fault model)**:**
  - good and bad state of the channel
  - Bit errors occur in the bad state



[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Gilbert-Elliott, Implementation

- Not efficient: simulating bit errors by applying the Gilbert-Elliott model bit by bit.
- Better: Determine the number of bits until the next state changes. This number is geometrically distributed.
- Even better: If the position of bit errors is not important, only the number, then it is more efficient to determine the number of bit errors in a certain time interval.

[H. Karl, Leistungsbewertung und Simulation, Uni Paderborn, 2007]

Lessons learned

- Simulation results are only an *estimation* of the true behaviour
- Pitfalls: bad random number generators, bad seed selection, insufficient simulation time
- Be careful when using aggregated information and interpreting results